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Idealizations, *Ceteris Paribus* Clauses, Idealizational Laws, and All That[1]

The aim of this paper is to clarify the role of idealizations in the formulation of scientific laws and scientific explanations by drawing on the results provided by philosophy of science in the last four decades or so.

I start with a differentiation between idealization, abstraction, and *ceteris paribus* clauses (section 1). Next, I offer a typology of idealizations and show how the various types can be made more precise by considering them from the point of view of their role and place in scientific laws and explanations (section 2).

In the second part of the paper, I provide, first, an initial model for such laws and explanations by drawing on (Nowak 1971, 1972, 1975) as well as on (Nowak, Nowakowa 2000) and then improve it further so that it corresponds more closely to the practice of empirical sciences (section 3). Based on this, I dispel the myth that the "end"-point of explanations in terms of idealizational laws should be factual laws in the sense assigned to them by logical empiricists such as Carl Hempel (section 4). Next, I further develop the model for idealized laws and explanations based on them and, finally, delineate boundaries beyond which it cannot be applied any more as a tool for the analysis and explication of idealizations as they occur in laws and explanations in empirical sciences (section 5).

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1. IDEALIZATIONS AS DIFFERENT FROM ABSTRACTIONS AND *CETERIS PARIBUS* CLAUSES

In order to set the stage for my methodological account of idealizations, it is appropriate to state what idealizations, at least in my view, are *not*. In everyday vernacular, idealization is often regarded as *abstraction* in the sense of a procedure by which we disregard certain features of the entities we are talking about. Even if such a view corresponds to everyday intuitions, there are certain features which distinguish idealization from abstraction (cf. Nowak 1975, Nowak, Nowakowa 2000, Jones 2009). In the latter we discount certain features of the entities under investigation as completely irrelevant to that investigation. Thus the performed abstraction is not mentioned any more in this framework — we are dealing with "mere omissions", where: "omission … is, so to speak, a matter of complete silence" (Jones 2009: 174-175). Later on, the abstracted features can be, for the purpose of a possible conceptual development and advancement, brought back into the investigation.

By contrast, in an idealization the entity under investigation is explicitly represented as temporally lacking a certain feature or several features and, in addition, at a certain point of the investigation these features can be, and usually are, brought back into the representation by cancelling that idealization. So, for example (cf. Nowak, Nowakowa 2000: 116), when one moves in description from an open capitalist economy to that of a capitalist economy, one performs an abstraction. But when on moves from the former to the description of a closed capitalist economy, one initially performs an idealization.

In addition, for the sake of precision, I would like to differentiate between, on the one hand, idealizations when they appear in statements of scientific laws and, on the other, the so-called *ceteris paribus* (*cp*) clauses — even if this distinction is not generally accepted (Kowalenko 2011).

Cp clauses are usually characterized as having the linguistic form "so long as other things are equal". But when they are part of statements of scientific laws, these statements are plagued by indefiniteness and epistemic vagueness. Idealizations do not display indefiniteness or epistemic vagueness because:

> the problem of *ceteris paribus* qualifications is distinct from the problem of idealizations. Often the idealization can be stated in a precise closed form (e.g. the ideal gas law assumes that the gas molecules have no volume and interact only by contact). Here the problem is not in saying precisely what is involved in the idealization but in relating it to the real world which is not ideal. By contrast, many cp laws claim to be about unidealized real world situations but make indefinite claims about these situations (Earman, Roberts 1999: 457).

Nowak (2000: 213-218) characterizes the role of *ceteris paribus* clauses in idealizational laws in a different way. Let the set P_F be the space of those magnitudes which are essential to the phenomenon F under investigation, and let $I(P_F)$ be the image of that space as understood by the respective scientists, that is, the class of factors they view as essential to *F*. Next, Nowak introduces the symbol "Q^F" which stands for the difference between these two sets in the sense of a set of disturbances of *F* from the perspective of the scientists. Finally, let there be a set of *m*-disturbances which deviate, once taken together, from *F* not more than by *m*; this set is designated by "Q_m^F".

By means of the latter set, Nowak can characterize *ceteris paribus* clauses as given in scientific laws by means of the formula "$Q_m^F(x) = 0$", which reads as follows: "All factors from the set Q_m^F equal zero for the object *x*". It states that factors outside the image of essential factors *F*, i.e. factors not taken into account by a scientist, do not exert any influence on *F* (Nowak, Nowakowa 2000: 215). Below, I will show how this understanding of *ceteris paribus* clauses leads Nowak to the issue of approximation as involved in the process of scientific explanation.

So far, idealization has been outlined in a negative way, namely, as different from both abstractions and *cp* clauses. Let me now turn to its positive description by differentiating between several types of idealization.

2. IDEALIZATIONS IN EMPIRICAL SCIENCES — A TYPOLOGY

I differentiate between the following three types of idealization.[2] First, *construct idealization* is performed in the course of the conceptual representation of the entities under investigation so that complex real world situations can be subjected to conceptual treatment. Here, certain aspects of the complex situation are deliberately set aside so as to obtain its simplification which can be treated conceptually. Frequently, construct idealizations have to be performed at the initial stage because the conceptual treatment of the disregarded aspects is not yet available. This also determines the "inverse" procedure of de-idealization, when those aspects are brought back into the conceptual treatment.

Second, there is *material idealization*: the respective conceptual system (even when several de-idealization procedures have been already performed) can be used as a conceptual material for further treatments of complex real-world situations — by applying the procedures of idealization/de-idealization once new aspects of these situations come to the fore and attempts at their conceptual treatment are made. As I will show below, a crucial role is played here by the discovery of previously unknown (so-called "hidden") idealizations.

Third, there are *causal subjunctive idealizations* which enable simplification of complex causal situations by, initially, leaving out certain causal factors which, for the time being, are viewed as secondary with respect to other factors or even as impediments for their actions. The de-idealization procedure should then consist in in-

[2] Here I draw partially on (McMullin 1985) but leave out the mathematical type of idealization, which provides a broader context in which the types of idealization presented here could be carried out. For another typology, see Weisberg 2007; for its extension, see Rohwer, Rice 2013.

tegration of the treatments of particular causal factors into a unified conceptual system providing knowledge about a network of causal relations.

The subjunctive aspect of this type of idealization consists in a search for an answer to the question: "What would happen if certain causal factors were, for the time being, not at work, while others, more relevant, were still at work?" This subjunctive dimension is then carried over to the de-idealization procedure which should find an answer to the question: "What would be the joint effect of several causal factors, some of them less relevant than the others, if they were simultaneously at work?"

As I will now try to show, Nowak developed an approach to the idealization/de-idealization procedures which enables, once it is further developed and improved, to cover all the above-listed types of idealizations/de-idealization procedures.

3. A MODEL OF IDEALIZATION

3.1. Nowak's model for idealization and de-idealization (concretization)

The reconstruction of the idealization/de-idealizational procedures employed in empirical sciences received its decisive impetus when philosophy of science turned to analyzing scientific laws such as the law for the period of swings of a simple (mathematical) pendulum or (Galileo's) law of free fall of a body in a nonresistant medium, which, it was claimed, are factually false in the sense that they are never fulfilled in the real world.

These laws can be seen as substitution instances of a universal conditional of the form:

> Given any *x*, if *x* fulfils certain conditions (which are never actually fulfilled), then if *x* is so-and-so, and *x* is so-and-so, and …, then *x* is such-and-such (cf. Barr 1971).

The antecedent here can be called an "ideal case", and idealizations like "friction acting on *x* equals zero" or "volume of *x* equals zero", which appear in the scientific laws, are viewed as expressions for *state variables* which in the ideal case are assigned zero values.

If F is a formula (a statement or a statement function) and a_1, a_2,…, a_n is a sequence of distinct individual variables, in order of occurrence in F, which are free in F, then $(\exists a_1)$, $(\exists a_2)$, …, $(\exists a_n) F$ is the existential closure of F. Formulas which describe ideal cases can be called "ideal conditions" for which the following holds:

> A formula is an ideal condition only if (i) F is a formula, (ii) the existential closure of F is factually false, (iii) F is not logically equivalent to a formula in which no state variable occurs, and (iv) the existential closure of F' is true, where F' is a formula obtained by replacing the values assigned to the state

variables in F with individual variables and existentially quantifying over each of these individual variables.

It is then possible to delineate an *idealized universal law* as follows:[3]

A statement L is an idealized universal law only if (i) it is a universal law, and (ii) its antecedent contains at least one ideal condition.

The merit of Nowak's approach as presented in (1971, 1972, 1975) and in (Nowak, Nowakowa 2000) is that it allows us to give a detailed methodological account of ideal conditions and thereby of both idealized laws and explanations by means of de-idealization based on these laws.

An idealizational law containing k idealizations, $L^{(k)}$, should have the following structure ("$\rightarrow$" stands for material implication):[4]

(1) $L^{(k)}: (x)[Gx \,\&\, p_1x = 0 \,\&\, p_2x = 0 \,\&\, \ldots \,\&\, p_kx = 0 \rightarrow Fx = f_k(H_1x, H_2x, \ldots, H_rx)]$

Here "0" denotes the value which the magnitude denoted by "p_i" ($i = 1,\ldots, k$) takes on, where no real individual from the range of "x" acquires this value; so "$p_ix = 0$" denotes an *idealizing assumption*, while "Gx" denotes a *realistic assumption* of formula (1), which specifies the type of entities (1) is about. In the consequent of (1) "F" denotes a magnitude with the status of *phenomenon* which, under given idealizations, depends on the magnitudes denoted by "H_1", …, "H_r", which are the *principal*, the most significant factors for F, while the remaining magnitudes p_1, …, p_k, which are not taken into account in (1), are *secondary* factors for F.

If the set of secondary factors $\{p_1, \ldots, p_k\}$ displays an order of significance for F, then the sequence of laws derived by explanation stemming from a law of the form (1) constitutes what was labelled above as "de-idealization" and by Nowak — "concretization". It involves an ordered sequence of both gradual abandonment of idealizations and gradual modification of the functional dependence between, on the one hand, the phenomenon F and, on the other, the factors H_1, …, H_r, p_1, …, p_k. This is expressed by Nowak as follows ("$p_ix \neq 0$" states that the i-th secondary factor acquires values different from zero, that is, it is at work and thus has an impact on F):

(2) $L^{(k-1)}: (x)[Gx \,\&\, p_1x \neq 0 \,\&\, \ldots \,\&\, p_kx = 0 \rightarrow Fx = f_{k-1}(H_1x, H_2x, \ldots, H_rx, p_1x)]$

....

$L^{(0)}: (x)[Gx \,\&\, p_1x \neq 0 \,\&\, \ldots \,\&\, p_kx \neq 0 \rightarrow Fx = f_0(H_1x, H_2x, \ldots, H_rx, p_1x, \ldots, p_kx)]$

Based on the law of the type $L^{(0)}$ one can, finally, perform the explanation of the individual phenomenon, *Fin*, by bringing in individual conditions, $Cind_{1\text{-}s}$, where the

[3] For the sake of simplicity, in this paper I deal only with laws which are universal in their nature, as opposed to statistical laws.

[4] For the semantics of expressions in (1), see Nowak 1972: 536.

derivation of *Fin* from $L^{(0)}$ is performed by deductive subsumption in the sense of Hempel's D-N model.

According to Nowak's approach to *ceteris paribus* clauses presented above, an idealizational law involving such a clause should have the following structure (Nowak, Nowakowa 2000: 215):

$_{cp}L^{(k)}$: $(x)[Gx \,\&\, p_1x = 0 \,\&\, p_2x = 0 \,\&\, \ldots \,\&\, p_kx = 0 \,\&\, Q_m^F(x) = 0 \rightarrow Fx = f_k(H_1x, H_2x, \ldots, H_rx)]$

Thus scientific explanation leads, ultimately, to a type of law whose structure is as follows ("$E_m^F(x) \neq 0$" stands for the negation of "$Q_m^F(x) = 0$"; "$\approx_m$" denotes approximation up to the certain threshold *m*; cf. Nowak, Nowakowa 2000: 216):

$L^{(0)}$: $(x)[Gx \,\&\, p_1x \neq 0 \,\&\, \ldots \,\&\, p_kx \neq 0 \,\&\, E_m^F(x) \neq 0 \rightarrow Fx \approx_m f_0(H_1x, H_2x, \ldots, H_rx, p_1x, \ldots, p_kx)]$

The crucial idea expressed here is that approximation is at work, because the joint impact of conditions $p_1 \ldots p_k$ is responsible only for deviations up to a certain threshold *m*, while the impact of the secondary factors expressed as "$E_m^F(x) \neq 0$" on *F* is not known (Nowak 1977: 137).

3.2. An improved model for an idealized type of law

Nowak's reconstruction of the structure of an idealized type of law as given in (1) can be further improved once one takes into account the practice of empirical science. What I have in mind is that the idealization clauses involved in scientific laws need not state that the secondary factors p_i always equal zero. It is very often the case that the secondary factors acquire a non-zero but still constant value or values from a certain limited interval. So, for example, in the case of the simple (mathematical) pendulum, it is supposed that the angle α of its deviation acquires in the course of the swings values from the interval <0°, 3°>, so that in the course of the derivation of the law for this pendulum one can suppose that $\sin \alpha = \alpha$. In a similar manner, in the case of Gay-Lussac's law stating the relation between the volume and temperature of an ideal gas, it is supposed that the pressure of the gas and the amount of gas are held at constant values (Chwalisz 1979).

A further revision and improvement of Nowak's reconstruction of the structure of the idealized type of law can be performed if one consequently draws on the meaning of his terminology. The phenomenon *F* under investigation, as symbolized in sequence (2), should have, according to him, the status of *manifestation* with respect to the primary and secondary factors (Nowak 1975: 27), while the latter constitute the *essential structure* with respect to *F* (Nowak, Nowakowa 2000: 120). To be more precise: for *F* as it appears in (1), its essential structure is the set $\{H_1, H_2, \ldots, H_r\}$, while for *F* as it appears in $L^{(k-1)}$, the essential structure is the set $\{H_1, H_2,$

…, H_r, p_1} and, finally, for F as it is given in $L^{(0)}$, its essential structure is the set {H_1, H_2, …, H_r, p_1, …, p_k}.

Thus, once the secondary factors are already at work, their central role in the process of gradual concretization needs to be expressed in the derivation of the corresponding phenomena as manifestations. When these factors, denoted by Nowak as "p_1", …, "p_k", are gradually introduced, they modify the initial manifestation F as it is stated in $L^{(k)}$. Accordingly, we symbolize this initial manifestation as "$F^{(k)}$", while the others, derived by gradual concretization — as "$F^{(k-1)}$", …, "$F^{(0)}$"; I also replace Nowak's "secondary factors" with "modification conditions", which I symbolize as "$Cmod_i$" and which can acquire not only zero values but also a certain constant value or values from a limited interval of values; these values are represented as "d_i".

With all this in mind, I restate Nowak's scheme for (1) as well as the structure of laws from the above given sequence (2) of concretized laws as follows:[5]

(3) $L^{(k)}$: $(x)[Ux \;\&\; Cmod_{1\text{-}k}x = d_{1\text{-}k} \rightarrow F^{(k)}x = f_k(Hx)]$

(4) $L^{(k-1)}$: $(x)[Ux \;\&\; Cmod_1 x \neq d_1 \;\&\; Cmod_{2\text{-}k}x = d_{2\text{-}k} \rightarrow F^{(k-1)}x = f_{k-1}(Hx, Cmod_1 x)]$

....

$L^{(0)}$: $(x)[Ux \;\&\; Cmod_{1\text{-}k}x \neq d_{1\text{-}k} \rightarrow F^{(0)}x = f_0(Hx, Cmod_1 x, \ldots, Cmod_k x)]$

Based on the reconstruction of the method of explanation by gradual concretization which starts from a scientific law of the form (3), one also arrives at a more detailed understanding of the course taken by scientific explanation.

If the aim of explanation is to derive an already known law, then explanation involves in fact two interlinked steps: first, subsumption of this law's universe of discourse under the universe of discourse of the law of the type $L^{(k)}$ and, second, a sequence of gradual concretization which should, ultimately, yield a law which is seen as the explanation of the initial law.

As an example of such a two-step procedure, consider the explanation of the regularity law according to which bodies on an inclined plane slide down in such a way that the distance they cover varies with the angle of the inclination.

The point of departure of explanation is the second dynamic law of Newtonian mechanics, which for the force of gravity has the following form ("O" stands for an object with a non-zero mass, the first idealization states that the body with mass m with an acceleration g due to the impact of force F of gravity has a zero volume, and the second idealization states that it moves in a nonresistant medium):

(5) $(x)(Ox \;\&\; Cmod_{1,2}x = d_{1,2} \rightarrow Fx = mxg^{(2)}x)$

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[5] Here "U" stands for the universe of discourse, that is the set of entities for which the law is stated; "$d_{1\text{-}k}$" for the conjunction "d_1 & … & d_k" and "$Cmod_{1\text{-}k}$" for the conjunction of modification conditions $Cmod_1$ & … & $Cmod_k$; for the sake of simplicity I assume that only one principal factor is given.

Next, one performs a *thought-reconstruction* to the effect that the sliding body turns into a mass-point sliding down the inclined plane, that is, an idealized thought-object, and then, for this type of entity, one states the equation $m\mathrm{d}v/\mathrm{d}t = mgsin\ \alpha$, where the expression on the right side stands for the component of the force of gravity acting on the mass-point sliding down an inclined plane with α as the angle of inclination. From this equation, together with its universe of discourse and the stated idealizations, it is possible to derive the following law for the distance covered by the sliding mass-point:

(6) $\mathrm{L}^{(4)}$: $(O'x\ \&\ Cmod_{1,2,3,4}\, x = d_{1,2,3,4} \rightarrow s^{(4)}x = g^{(2)}xt^2xsin\ \alpha x)$

where 'O'' stands for the mass-point sliding on an inclined plane; "$Cmod_{1,2} = d_{1,2}$" stands for the conjunction of the two idealizations given already in (5); "$Cmod_3 = d_3$" stands for the idealization that the sliding mass-point starts its sliding from rest (i.e. its initial velocity equals zero); "$Cmod_4 = d_4$" stands for the idealization that there is no force of friction decreasing the accelerated motion of the mass-point along the inclined plane; "$g^{(2)}$" stands for acceleration due to the force of gravitation under the two idealizations transposed from (5).

Thanks to (6), one can explain, by abandoning idealization $Cmod_4 = d_4$, how the covered distance changes once friction is at work (friction is proportional to $gcos\ \alpha$):

(7) $L^{(3)}$: $(x)[O'x\ \&\ Cmod_{1,2,3}x = d_{1,2,3}\ \&\ Cmod_4\ x \neq d_4 \rightarrow s^{(3)}x = g^{(2)}xt_{\text{¸}}x(sin\ \alpha x - cos\ \alpha x)]$

This example shows that the process of explanation comprising subsumption and gradual concretization involves non-eliminable heuristic components. The first one is given in the transformation of the universe for which the law to be explained was initially stated so that after transformation it can be viewed as falling under the universe of discourse of the explanans-law. The second one is given in theoretical rendering of the impact of the respective modification condition on the relation between the respective phenomenon and the principal factor. Only once this relation is understood can it be expressed in the form of a functional relation.

What has also changed is the very explanandum. At first the only known element was the regularity that bodies on an inclined plane slide down in such a way that the distance they cover when sliding varies with the angle of inclination of the plane. Accordingly, what was derived was a law of the form (3). Thus the very explanandum underwent a profound restatement and reconstruction in the course of its derivation by explanation.

3.3. The case of "hidden" idealizations

In Nowak's approach, as shown above, approximation has to be at work in explanation when scientists know that certain secondary factors are at work but do not as yet know their impact on the phenomenon under investigation.

A situation different from the latter can be methodologically understood once we return to the above reconstruction of the process of explanation as involving both gradual concretization and deductive subsumption. It was presupposed that this process starts from a law in which all the modification conditions necessary for the explanation by gradual concretization are already known before the explanation of the individual phenomenon is performed. The only task left here is that of performing concretizations yielding $L^{(k\text{-}1)}$, $L^{(k\text{-}2)}$, …, $L^{(0)}$, and of performing an explanation of the individual phenomenon *Fin* by deductive subsumption from $L^{(0)}$ by introducing the respective individual conditions. We express this situation as follows ("⊣" stands for gradual concretization, $Cmod_i$ indicates that the idealizations $Cmod_i = d_i$ are cancelled inside $L^{(k)}$, …, $L^{(1)}$; "⊢" stands for entailment):

(8) $L^{(k)}$ & $Cmod_1 \dashv L^{(k\text{-}1)} \ldots \dashv L^{(0)}$ & $Cind_{1\text{-}s} \vdash Fin$

↑ task ↑ task ↑ task

These tasks, however, can fail once the known modification conditions do not enable performing the concretizations leading to $L^{(0)}$ and, thereby, the move from $L^{(0)}$ to *Fin* cannot be made. The reason is that very often certain modification conditions necessary for performing the concretization procedures are as yet unknown. In such a case one speaks of "hidden idealization".

Nowakowa has dealt with such a case from the point of view of philosophy of science (Nowakowa 1974, Nowak, Nowakowa 2000: 187-189). Her reconstruction of that failure and of a possible remedy can be expressed as follows (where "⊥" stands for gradual concretization, "|→" for the introduction of an additional idealization, "Δ" is a subscript indicates the failure of explanation).[6]

[6] For the sake of simplicity, I disregard the fact that scientists can first realize the failure once their attempt at the explanation of an already known *individual phenomenon* has failed (not only once their attempt at the explanation of an already known *law* has failed).

$$
\begin{array}{lllll}
L^{(k)} & \mid\!\rightarrow & L^{(k+1)} & \mid\!\rightarrow & L^{(k+2)} \\
\perp & & \perp & & \perp \\
L^{(k\text{-}1)} & & L^{(k)} & & L^{(k+1)} \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
\perp & & \perp & & \perp \\
{L^{(0)}}_{\Delta} & & L^{(1)} & & L^{(2)} \\
 & & \perp & & \perp \\
 & & {L^{(0)}}_{\Delta} & & L^{(1)} \\
 & & & & \perp \\
 & & & & L^{(0)}
\end{array}
$$

Figure 1. Failure of explanation leading to discovery of hidden idealizations

The idea here is that once a failure of explanation is detected at the "end"-point of the explanatory sequence, then the source of this failure is located in the law of type $L^{(k)}$ in the left column (the same holds for $L^{(k+1)}$ in the central column) at the top of this sequence, and it is found out that this law cannot be used for explanation of the respective law or individual phenomenon because it lacked a certain modification condition. Once the missing condition has been identified, it is integrated into the law of the type $L^{(k)}$ ($L^{(k+1)}$) so that it turns into the law of the type $L^{(k+1)}$ ($L^{(k+2)}$) from which the explanation procedures can start again.

3.4. Explanation involving both introduction and cancellation of idealizations

Nowak's approach to idealization/concretization can also be further developed once we realize that in explanation procedures cancellation of idealizations can go hand in hand with introductions of additional idealizations. This was the case, for example, in Schrödinger's derivation of the various forms of the wave equation in his *Mitteilung* I through IV (1926a, 1926b, 1926c, 1926d).

Schrödinger stated in the first *Mitteilung* the initial form of his wave equation $\Delta\psi + \frac{2m}{K^2}(E + \frac{e^2}{r})\psi = 0$ for the hydrogen atom under two idealizing assumptions: the speed v of its electron with charge e and mass m is much smaller than the speed of light c, $\frac{v}{c} = 0$, and the atom is not disturbed by external forces, $Fe = 0$.[7] The equation, once taken together with these idealizations, yields the following scientific law ("*S*" stands for a one-electron system):

(9) $L^{(2)}$: $(x)[Sx \;\&\; Fex = 0 \;\&\; \dfrac{vx}{c} = 0 \rightarrow \Delta_x \psi x + \dfrac{2m}{K^2}(Ex + \dfrac{ex^2}{rx})\psi x = 0]$

[7] Here "ψ" stands for the wave function, $K = h/2\pi$, where h is Planck's constant, "E" stands for energy of the one-electron system.

The scientific law of the form (9) is then generalized in the second *Mitteilung* by introducing the magnitude V standing for the potential for which the idealization $\frac{\partial V}{\partial t} = 0$ should hold:

$$(10) \quad L^{(3)}: (x)[Sx \ \& \ Fex = 0 \ \& \ \frac{vx}{c} = 0 \ \& \ \frac{\partial V}{\partial t} = 0 \rightarrow \Delta_x \psi x + \frac{8\pi^2}{h^2}(Ex - Vx)\psi x = 0]$$

In the third *Mitteilung*, Schrödinger abandons the first idealization in (9) and at the same time introduces magnitudes characterizing two external forces, the magnetic, with the strength F_{em}, and the electric, with the strength F_{ee}, so that the physical system is subjected only to the latter, that is $F_{ee} \neq 0$, while $F_{em} = 0$, and the system under consideration is again a one-electron system. For such a system the following law holds:

$$(11) \quad L^{(3)}: (x)[Sx \ \& \ F_{em}x = 0 \ \& \ \frac{vx}{c} = 0 \ \& \ F_{ee}x \neq 0 \rightarrow \Delta_x \psi x + \frac{2m}{K^2}(Ex + \frac{ex^2}{rx} - eF_{ee}zx)\psi x = 0]$$

In the fourth *Mitteilung* Schrödinger departs from law (10) by giving up its third idealization and derives the following law:

$$(12) \quad L^{(2)}: (x)[Sx \ \& \ Fex = 0 \ \& \ \frac{vx}{c} = 0 \ \& \ \frac{\partial Vx}{\partial tx} \neq 0 \rightarrow (\Delta_x - \frac{8\pi^2}{h^2}Vx)^2 \psi x + \frac{16\pi^2}{h^2}\frac{\partial^2 \psi x}{\partial tx^2} = 0]$$

In the last step, he abandons the first and second idealizations in (11) and so arrives at the relativistic-magnetic generalization of the wave equation and thus at a law of the type $L^{(0)}$.

This sequence can, from the point of view of philosophy of science, be generalized as follows (letters "*l*," "*m*," "*n*," "*o*", and "*p*" stand, like "*k*", for positive integers or zero; the combination of the arrow-symbol with that of "⊥" stands for simultaneous cancellation and introduction of idealizations):

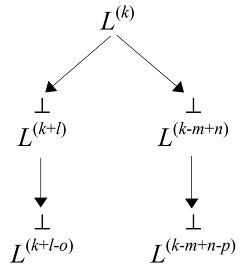

Figure 2. Sequences of explanation procedures combining introduction and abandonment of idealizations

Figure 2 shows that explanation by gradual concretization starting from a law of the type $L^{(k)}$ need not take its course just in one ordered sequence. Given that secondary factors are not ordered according to their importance, the explanation can lead to a branched sequence of laws.

In the case of gradual derivation of the various forms of the wave equation, the reason for such branching is that the modification conditions are introduced into the laws, first in an idealized form and then in an de-idealized form, by drawing on conceptual resources of other physical theories, that is, what is usually called "theories in the background", namely, mechanics (the magnitude V), theory of electromagnetism (magnitudes F_{ee} an F_{em}), and relativity theory (the ratio of v to c; first as $\frac{v}{c} = 0$ and then as $\frac{v}{c} \neq 0$).

The possible existence of a branching sequence of concretized laws leads to the following general question about the method of explanation by gradual concretization: *When does this method — if at least two idealization conditions are involved in the law of the type* $L^{(k)}$ *— lead to the same laws and when not, presupposing here of course, that the set of these conditions displays no inherent order of importance with respect to its elements?*

As an example, consider a law with just two idealizations. Explanation based on it can take the following two courses (the lower index indicates which idealization has already been abandoned — the first or the second, as given in $L^{(k)}$):

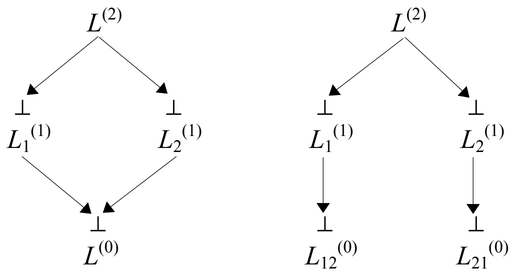

Figure 3. Different courses taken by explanation by means of the method of gradual concretization

While the left diagram expresses the fact that the order in which the concretization steps are performed is not important for the end-result of the application of the method of gradual concretization, the diagram on the right indicates that this order is relevant: the law of the type $L_{12}{}^{(0)}$ is not identical with that of $L_{21}{}^{(0)}$.

Their possible non-identity could be captured in terms of concepts developed in semantics and in the particular empirical science which would state the respective idealization laws. From the semantic perspective, the difference between those two laws could be approached by reflecting on the difference between the functions given in their consequents: $F^{(0)} = f_0(H, Cmod_1, \ldots, Cmod_2)$ and $F^{(0)} = f_0(H, Cmod_2x, \ldots, Cmod_1)$. The empirical science could bring in its understanding of the objects for which the laws in the sequences are stated: the possible identity and non-identity of the laws obtained by gradual concretization can depend on the nature of these objects. Thus, for instance, in social sciences, which explain sequences of human actions and their results, the results of such sequences might vary depending on the order of actions. It is a task for future research to perform such a combined semantic-cum-empirical-science explanation.

Let me now ask whether, and if so how, Nowak's reconstruction of the structure of an idealized type of law and of the structure of the procedure of de-idealization as concretization fits the typology of idealizations given above.

As to the *construct* idealization, Nowak's approach allows us to reconstruct how certain aspects of the entities under investigation (from a set G) are initially simplified so as to express, at first, the relation between their phenomenal characteristic of the type F and their principal factor, and then how the overall characterization of F is gradually developed by bringing "back" in, step by step, the conditions relevant to this phenomenon.

As to *material* idealization, it is very often the case, as we have seen, that not all modification conditions are known in advance. Still, a theory, understood as a sequence of laws of types $L^{(k)}$, $L^{(k-1)}$, …, $L^{(k-j)}$, not only can provide means for its own testing, but also, once it fails this test, the means of its own development, that is, conceptual resources to trigger and accomplish what McMullin (1985: 264) describes as the "process of self-correction and imaginative extension".

The *causal* type of idealization can be seen as captured by Nowak's approach once one gives a causal interpretation to the sequence of relations between the investigated phenomenon and the described principal factor and secondary factors. These relations can express the relation between a cause and its effect, as it is in the case of $F^{(k)} = f_k(H)$, while the whole sequence $F^{(k-1)} = f_{k-1}(H, Cmod_k)$, …, $F^{(0)} = f_0(H, Cmod_k, \ldots, Cmod_1)$ stands for the gradual derivation of knowledge about a network of causal relations producing the respective phenomena $F^{(k-1)}$, …, $F^{(0)}$.

Finally, the *subjunctive* aspect of the causal type is encompassed by Nowak's approach in that it provides the conceptual means, in the framework of philosophy of science, to deal with the situation when scientists ask and search for an answer to the following type of question: *What laws would have been produced, given a certain law known to be at work, under the impact of certain causal factors* $Cmod_1$, …, $Cmod_j$, *also known to be at work, if other known factors had not been simultaneously at work, i.e.* $Cmod_k = 0$, …, $Cmod_p = 0$*?* I express this situation by the following scheme (where "???" stands for "laws as yet unknown"):

$$\frac{\begin{array}{l} L^{(k)} \\ Cmod_1, \ldots, Cmod_j, Cmod_k = 0, \ldots, Cmod_p = 0 \end{array}}{???}$$

We should distinguish this case from two other “as yet unknown” situations.[8] First: Which modification conditions could be at work and which not, given a law known to be operational, in order for certain laws L_1, L_2, …, L_s, already known to hold, to be produced? The corresponding scheme is:

$$\frac{\begin{array}{l} L^{(k)} \\ ??? \end{array}}{L_1, L_2, \ldots, L_s}$$

Second: *For certain laws* L_1, L_2,…, L_s *already known to hold, what laws and modification conditions could be at work in order for* L_1, L_2,…, L_s *to be produced?* The scheme here is:

$$\frac{\begin{array}{l} ??? \\ ??? \end{array}}{L_1, L_2, \ldots, L_s}$$

4. A DEFENSE AGAINST TWO OBJECTIONS

Let me now turn to two objections raised against Nowak’s approach to idealizational laws and their concretization.

4.1. Beyond the myth of the factual

In philosophical accounts of idealizations involved in scientific laws, theories, hypotheses, models, etc., the standard claim is that the latter are false because the idealizations involved in them make them false (Jones 2009: 174-175; Wayne 2011: 831) with respect to reality:

> Semantically, the outstanding characteristic of idealizations is that they literally describe nothing — there is no entity, or state of affairs, to which the idealization stands in a designatory or descriptive relation (Rudner 1966: 57).

The reason for this, it is claimed, is that “they fail to capture what is actually observed” (Hindriks 2013: 524) and:

> real world phenomena often contravene idealized hypotheses. These hypotheses seem to misrepresent phenomena and the source of falsity in each case seems to be […] the constituent idealizations for each hypotheses (Jones 2013: 85).

In more recent philosophical literature, the idealizational laws, theories, etc. with their “unrealistic” status (Hindriks 2013: 523) and lack of a “real domain” (Krajew-

[8] Here I draw partially on (Putnam 1974) and (Sintonen 2005).

ski 1977: 325) are contrasted with empirical laws or observed factual regularities, which are regarded as true and which satisfy two conditions: (i) they should be explained by the former in the sense that "theoretical laws derived from the [idealized] model give an approximate fit with empirical laws reporting on observation" (McMullin 1985: 264), and (ii) their "immediate warrant is observation or experiment" (McMullin 1985: 257).

Nowak also accepts these views, at least partially. He differentiates between an idealizational term and a factual term in such a way that the former's "content is partly due to idealizing and partly to realistic assumptions (in the light of a given knowledge)" (1975: 25), while the latter's "content is characterized by realistic assumptions only" (1975: 25). This leads him to the claim that a law of the type $L^{(0)}$ should be a *factual statement*, since it is the "end"-point of explanation by gradual concretization and so does not contain any idealizations (Nowak, Nowakowa 2000: 118, 129). At the same time, however, this type of law is supposed to include statements such as "Every raven is black" (Nowak 1975: 26).

Nowak distinguishes between factual and idealized laws also by differentiating between types of object by which they are satisfied:

> In general it could be said that idealizational laws differ from factual laws by being non-vacuously satisifed by ideal types and not by real objects (Nowak 1971: 40-41).

It is worth noting, however, that Nowak's position differs from the view, inspired by logical empiricism, that a factual statement is of purely perceptual (non-theoretical) nature:

> I will call *factual* a statement whose fulfilment (non-vacuous in the case of conditionals) is not excluded. Attention must be paid to the fact that factual statements, conceived of in this way, can be both observational and theoretical (1971: 40).

Once this last claim is discounted, the above presented claims of Nowak, as well as the above claims put forward in contemporary philosophical literature about laws, theories, etc. involving idealizations, lead to Werner Diederich's critique of the method of explanation by gradual concretization. He makes the following observation about the explanation of the individual phenomenon in the explanatory sequence represented above in (11):

> No doubt, such an explanation would be totally in accord with the Hempelian pattern; […] the last part of Nowak's scheme $L^{(0)}$ & $Cind_{1\text{-}s} \vdash Find$ simply satisfies Hempelian criteria for explanation. On the other hand, none of the idealizing laws $L^{(1)}$, …, $L^{(k)}$ instead of $L^{(0)}$ would to the job. Nor would there be any help at all for a Hempelian explanation once $L^{(0)}$ is at hand (Diederich 1994: 259; I have replaced Diederich's notation with that given in (8)).

What Diederich claims here is that once a law of the type $L^{(0)}$ is given — understood as a factual type of law in the logico-positivist sense, that is, as a perceptually observed regularity — then the whole sequence of laws leading to $L^{(0)}$ is redundant;

this type of law by itself would do the (D-N) explanatory job represented as $L^{(0)}$ & $Cind_{1\text{-}s} \vdash Fin$.

One may respond to this critique, as well as to the above claims about the antirealistic nature of idealizational laws, as follows. The law of the type $L^{(0)}$, which should be the "end"-point of explanation by gradual concretization, is not a factual type of law in the Hempelian sense, that is, pertaining to a regularity based on perceptual observations, for example, "Every raven is black". On the contrary, it has a highly *theoretical* nature because in its antecedent it contains terms referring to modification conditions and in its consequent it contains terms referring to the interaction between the principal factor and the modification conditions as secondary factors, as well as an expression of the relation between all these factors and the phenomenon of the type $F^{(0)}$.

Where does this theoretical nature of the law of the type $L^{(0)}$ come from? It comes, partially, from the law of the type $L^{(k)}$, in which the modification conditions and the principal factors are stated, namely, they are transferred to $L^{(0)}$ from $L^{(k)}$ via $L^{(k\text{-}1)}$, …, $L^{(1)}$. This theoretical nature also stems from the process of gradual concretization by means of which that transfer is performed and by means of which, eventually, we arrive at the consequent of $L^{(0)}$. The whole sequence $L^{(k)}$, $L^{(k\text{-}1)}$, …, $L^{(0)}$ is theoretical "from the top to the bottom".

Given this theoretical nature of all the components in the sequence $L^{(k)}$, $L^{(k\text{-}1)}$, …, $L^{(0)}$, it is readily seen that the further one moves in the process of explanation from $L^{(k)}$ to $L^{(k\text{-}1)}$, …, to $L^{(0)}$, the more theoretical (theory-laden) the laws in this sequence become. *In the process of explanation by gradual concretization one moves from a theoretically abstract (poor) piece/body of knowledge to more and more theoretically concrete (richer) pieces/bodies of knowledge*. Thus the process of explanation by gradual concretization lacks the character of factualization in the sense that at the "end"-point of the explanation sequence one obtains a factual type of law in a logico-positivist sense.

Once this view on the process of explanation by gradual concretization is accepted, it is also possible to find a correct answer to the question about the type of objects by which the idealizational laws of the types $L^{(k)}$, …, $L^{(0)}$ are non-vacuously satisfied.

A law of the type $L^{(k)}$ is non-vacuously satisfied by a theoretical representation of real objects of a certain kind which are considered — by means of this representation — as a relation holding (under k idealizations) between the phenomenon of the type $F^{(k)}$ and the principal factor. Laws of the types $L^{(k\text{-}1)}$, …, $L^{(0)}$, in turn, are non-vacuously satisfied by real objects of a certain kind, where by "real" one should not understand apparent facts graspable by perceptual observation — as claimed by logical positivists — and expressed by the so-called "factual" laws of science; instead, real objects are theoretically represented as a relation which holds (under k-1, k-2, …, 1, and finally under 0 idealizations) between, on the one hand, the phenomena

$F^{(k)}$, …, $F^{(0)}$ and, on the other, the mutual action and interaction between the principal factor and the modification conditions.

The claim, very frequently encountered in the more recent philosophy of science, that idealizational laws are not satisfied by real objects holds only if by "real objects" one understands the apparent ("surface") characteristics of these objects that can be perceptually observed. Idealizational laws of the types $L^{(k)}$, …, $L^{(0)}$, however, do not deal with *appearances*, because they are not stated for them at all. They only deal with a principal factor and its relation (under given idealizations and acting modification conditions) to *manifestation* $F^{(k)}$, …, $F^{(0)}$ of this factor.

4.2. The epistemic priority of gradual concretization as compared to entailment

Diederich criticizes Nowak's approach to concretization also with respect to the presence of the sequence $L^{(k)}$, … , $L^{(1)}$ in this explanation, because:

> once $L^{(0)}$ is at hand […] they are redundant: they are a consequence of all less idealizing laws; […] if in the totally concretized law […] all disturbing terms vanish, you get back the consequence of the most idealized law (and by implication this law itself) (Diederich 1994: 259; the original notation replaced with my own).

With respect to the explanation procedures represented in sequence (8) given above, this means that one should obtain the following sequence of entailments:

(13) $L^{(0)} \,\&\, Cmod_1 = d_1 \vdash L^{(1)} \,\&\, Cmod_2 = d_2 \vdash L^{(2)} \ldots \vdash L^{(k\text{-}1)} \,\&\, Cmod_k = d_k \vdash L^{(k)}$

This sequence, according to Diederich, should make it clear that the whole procedure of explanation by gradual concretization has no epistemic value of its own; instead the relation of entailment should take centre stage.

Diederich's critique seems epistemologically unsound for two reasons. First, in order to obtain (13), the law of the type $L^{(0)}$ has to be given; but $L^{(0)}$ can be obtained only by performing the sequence of gradual concretization starting from a law of the type $L^{(k)}$. The derivation of $L^{(0)}$ is an epistemic accomplishment; one cannot derive it simply by bypassing the sequence of gradual concretizations.

Second, it follows that the laws from the sequence $L^{(k)}$, $L^{(k\text{-}1)}$, … , $L^{(1)}$ have *epistemic* (*cognitive*) priority over $L^{(0)}$. It becomes clear once we consider the above characterization of the method of explanation by gradual concretization, namely, that by its application the laws involved in the concretization sequence grow more and more theoretical (theory-laden). This means that an explanation accomplished by this method leads to *an increase in knowledge*. Consequently, the "backward" idealization of modification conditions, that is, in our notation, a reintroduction of idealizations $Cmod_i = d_i$, which have been already cancelled, is possible, but *epistemically completely unproductive*. In order to perform a derivation by entailment of the laws of the type $L^{(1)}$, $L^{(2)}$, …, $L^{(k)}$ as represented in (13), one had first to accomplish gradual concretization starting from $L^{(k)}$. Thus, even if (13) holds — given that the derivation by entailment is understood as an inverse derivation with respect to that obtained by gradual concretization — *from the point of view of the growth of knowledge, the procedure of gradual concretization has epistemic priority over derivation by entailment*.

It is also worth noting that in the sequence of concretizations represented in (4), as opposed to the sequence of entailments represented in (13), an irreplaceable role is played by heuristics, in the sense outlined above.

5. CONCLUSION — LIMITATIONS OF NOWAK'S MODEL OF IDEALIZATION

Let me conclude by pointing out certain shortcomings of Nowak's model of idealization, which, due to fundamental characteristics of his account, cannot be overcome. These limitations become evident once we pose the following three questions about Nowak's model.

Question 1: Why does the principal factor manifest itself as it does at all?

As shown above, the whole methodological reconstruction of the explanation procedure by gradual concretization understood as a derivation of the manifestations $F^{(k-1)}$, …, $F^{(0)}$ is based on the assumption that the principal factor, once the modification conditions 1 through k are not at work, manifests itself as $F^{(k)}$. But why does that factor manifest itself at all in a manifestation of the type F and not as a manifestation of, say, the type I? This question pertains to the issue of the existence of a manifestation as such, that is, pertains to its *quality*. There is not even a hint of an answer to this question in Nowak's approach; he simply presupposes that a manifestation of this type is given. In this paper I will not propose any answer to this question but just try to indicate its importance for the methodological analysis of science.

The principal factor produces a manifestation of the type (quality) F and not, say, I, not because all the modification conditions equal zero. In fact, the opposite is true. It is because the principal factor produces a manifestation of the type (quality) F, that it manifests itself — once the modification conditions are not at work — as $F^{(k)}$, and, once they are at work, as $F^{(k-1)}$, …, $F^{(0)}$. The differences between manifestations $F^{(k)}$, $F^{(k-1)}$, …, $F^{(0)}$ are quantitative differences between manifestations sharing the same type (quality) F of manifestation.

Suppose that — in the framework of an empirical science — we fail to know why a certain principal factor produces a certain type (quality) of manifestation. What effect would this lack of knowledge have? Since the claim that the principal factor manifests itself under the given k idealizations as $F^{(k)}$ is based on the assumption that it manifests itself from the point of view of a type of manifestation as F, then, if no justification for the production of this type of manifestation is given, the whole explanation of the quantitative deviations of $F^{(k-1)}$, …, $F^{(0)}$ from $F^{(k)}$ would lack a proper foundation. This lack would then penetrate into the whole explanation sequence from $L^{(k)}$ to $L^{k-1)}$, …, to $L^{(0)}$.

That this is so becomes evident once we realize that the very sequence of concretizations serves the purpose of explaining why the manifestations $F^{(k-1)}$, …, $F^{(0)}$ deviate *quantitatively* from the quantitative determination of manifestation of the type F expressed by $F^{(k)}$. In this explanation the latter is, with respect to the former, a quantitatively characterized centre around which the other quantitative values of all manifestations of the type (quality) F oscillate.

Question 2: How does one acquire knowledge about the principal factor?

Nowak gives the following methodological characterization of the road to the formulation of the idealizational type of law:

> The method of idealization consists basically in three steps: the adoption of idealizing assumption, the forming of idealizational hypotheses, and their concretization (Nowak, Nowakowa 2000: 10).

And with respect to the hierarchical order of the levels of essential structure $\{H\}$, $\{H, Cmod_1\}$, …, $\{H, Cmod_1, …, Cmod_k\}$ given in the sequence of laws of the type $L^{(k)}$, …, $L^{(0)}$ setting up a scientific theory, he declares that:

> [scientific] theory begins with reconstruction, in the form of the initial law, of the dependence holding on the first level, and the further concretizations of the law reconstruct more realistic dependencies holding on the subsequent levels of the structure (Nowak, Nowakowa 2000: 121).

Nowak, therefore, says nothing about the road to the formulation of the law of the type $L^{(k)}$. He just presupposes that scientists are able, somehow, to grasp directly the relation between the phenomenon $F^{(k)}$ and the principal factor H stated in that type of law.

It is also worth noting here that Nowak justifies his views on the essential structure of a phenomenon by referring to Hegel:

> An approach to idealization […] could be termed neo-Hegelian as it refers to Hegel's idea that idealization […] consists in focusing on what is essential in a phenomenon; […] I shall label that conception the idealizational approach to science (Nowak, Nowakowa 2000: 5).

This suggests that Nowak missed a crucial aspect of Hegel's views (presented in the *Science of Logic*) about the road cognition takes if it wants to acquire knowledge about the essential structure underlying a phenomenon. What I have in mind is Hegel's reconstruction of the set of categories framing the road knowledge takes when it proceeds from the knowledge of phenomenon to what Hegel, inside the category cluster *Essence*, labels as grasping of the *ground* of the phenomenon. Hegel speaks here about the "retreat into the ground" and combines it with a reverse procedure of "coming out from the ground", so that one obtains a "retreat into the ground and coming out if it" (Hegel 1923: 103, 2010: 402).

In my view, the road from a phenomenon to its principal factor, in the sense of the ground of this phenomenon, is expressed in a scientific law of the following form (where $f_k(H)$ stands for a function of the principal factor):

(14) $L^{(k)}: (x)[Ux \ \& \ Cmod_{1\text{-}k}x = d_{1\text{-}k} \rightarrow f_k(Hx) = F^{(k)}x]$

In (14) the principal factor is grasped and at the same time quantified on the basis of an idealized phenomenon and its quantitative characterization. This grasping and quantification is illustrated, for example, by the second dynamic law of Newtonian mechanics as stated in (6).

As for the realism of a law based on (14), it is non-vacuously satisfied by the theoretical representation of real objects whose appearance under the given k idealization, $F^{(k)}$, has been used for the identification and quantification of its principal factor H. Thus it has, like the laws of the type $L^{(k)}$, …, $L^{(0)}$, a theoretical character; it does not pertain to perceptually observable phenomena.

A law of the form (14) can be used as explanatory basis of explanation by gradual concretization. At the basis of such a use lies the reversal of the relation of the type $f_k(H) = F^{(k)}$ as given in (14) into that of $F^{(k)} = f_k(H)$ as given in (3).[9] By such a reversal one obtains a law of the form (3), from which the explanation by gradual concretization can proceed. By means of the latter one obtains a sequence of laws given in sequence (4) above.

The whole sequence of laws expressed by (14), (3), and (4) can be regarded as a methodological explication of Hegel's phrase "retreat into the ground and coming out of it".

Question 3: Can a historical type of explanation be performed in the framework of Nowak's model?

Yet another type of limitation inherent in Nowak's approach to explanation can be brought to the fore once we turn to the issue of providing a historical type of explanation. In the contemporary philosophy of science, this type of explanation is very often understood as explanation of "the occurrence of some particular event or state of affairs by describing how it came to be" (Glennan 2010: 251).

Yet by a "historical explanation" one can also understand something very different, namely, the process by means of which a certain structure characterized by a principal factor, or a set of principal factors, underlying certain types of phenomena was transformed into a different structure characterized by a different principal factor, or a set of principal factors, producing other types of phenomena.

Once this is viewed as a possible task of explanation, yet another principal limitation of Nowak's model of explanation by gradual concretization becomes evident: at the very core of Nowak's model lies the assumption that in the course of explana-

[9] For a detailed reconstruction of this reversal, see Hanzel 2010.

tion all the components of the laws involved in explanation can be subjected to changes *except the universe of discourse and the principal factor*.

As can be seen from the formulas in (3) and (4), all the laws involved in the explanatory sequence pertain there to the entities of the same universe sharing the same principal factor. Thus the method of explanation by gradual concretization cannot be applied once the aim of a historical type of explanation is understood as a derivation of a law L_B from the law L_A, where the former is stated for entities from a universe U_A characterized by the principal factor H_A, while the latter is stated for entities from a universe U_B characterized by the principal factor H_B, and it is the case that $H_A \neq H_B$. The problem immanent to the historical explanation can be expressed as follows:

$$\frac{\begin{array}{l} L_A \\ ??? \end{array}}{L_B}$$

Here "???" stands for a type of conditions which are relevant for the existence of the principal (essential) factor, and this factor is the property which, once displayed by entities, makes them entities of a certain kind, that is, makes them belong to a certain universe of entities. What this type of conditions amounts to needs to be found out in the future; here I can only say that they differ both from modification conditions and individual conditions.

The fact that an idealizational type of law of the form given in (3) cannot be employed in a historical type of explanation in the sense explicated above has its roots in the assumption that the very existence of the principal factor is unconditioned. Only the respective manifestations are regarded as conditioned — by the action of modification conditions. This is not surprising given the fact that Nowak's reconstruction aimed only at a methodological explication of the $L^{(k)}$ type of law, that is of a type of law which would stand for the knowledge about the relation between the phenomenon $F^{(k)}$ and its principal factor H. This reconstruction completely bypasses the question of how the property of being an element of the set U is related to displaying the property H. This question, meaningful in itself, completely exceeds the conceptual capacities built into Nowak reconstruction of the structure of $L^{(k)}$. The search for an answer to this question is a task for future research.

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