1. DEFINITIONS

The Bundle Theory of Particulars (BT) refers in this paper to the view that:

(BT) particulars are complexes of properties (be that universals or tropes) which all stand in a contingent and primitive relation of coexemplification to each other (cf. Casullo 1988: 25).

The Substratum Theory (ST) refers to the view that:

(ST) particulars consist in a “bare particular” or “substratum” which bears the identity of that object and to which all the properties attach.

The Trope Theory (TT) refers to the view that:

(TT) qualities are particular (as opposed to universal) entities that are either simple (primitive) or composed of other tropes only. Resemblance and coexemplification are fundamental relations, with properties analyzed as resemblance classes of tropes (cf. Maurin 2001: 175-176).

An accidental property is one that could have been absent in a particular that actually has it; or a property that could have been had by a particular that actually lacks it. The “could have been” is understood counterfactually, that is, as a shorthand for “there is a possible world, such that…” (Mackie 2009).

Accidental properties stand in opposition to necessary properties: those that are essential; those that could not have possibly been lacking, or present otherwise than in the actual configuration.
The Principle of Identity of Indiscernibles refers to the view that:

\[(\text{PII})\] if some objects possess exactly the same properties, then they are one and the same object: \(\forall P (P(x) \equiv P(y)) \rightarrow x = y\).

2. RATIONALE AND SCOPE OF THE PAPER

There is a two-fold rationale for choosing the clash between BT and accidental properties as an interesting research project.

It is easy to show that BT does without postulating the troublesome property-less substratum: a notion that can be criticized on the grounds of empiricism as a quality-less and unknowable substance. Similarly, it is not clear whether the very notion of a property-less substratum is not self-defeating, since presumably we can find certain higher-order properties that should be attributed to the substratum: the property of being property-less or the property of being the base for numerical identity, if there are such properties. Second, while ST postulates two ultimate ontological categories (properties and substance), BT reduces particulars to properties and thus ends up with a one-category ontology.

Recapitulating arguments against ST, however, is not the aim of this paper. Here, I evaluate four arguments formulated against BT, namely: the argument from tautologous subject-predicate discourse, the argument from essentialism, the argument from the impossibility of diachronic change, and the argument from PII. In what follows, I offer a general strategy to deal with those and structurally similar problems and show how it answers the four objections.

3. THE PROBLEMS

3.1. Tautologous subject-predicate discourse

A class of objections aims to show that the language offered by BT is unsuitable for a satisfactory analysis of subject-predicate (S-P) sentences. One possible way of formulating this objection is to say that all true S-P sentences are tautologous, while all false S-P sentences are countertautologous. Another way of putting the objection is to say that all sentences about particulars are either necessarily true or necessarily false. If this were true, all particulars would exemplify all of their properties out of necessity, which means that no particular could either exemplify any other property than those it actually exemplifies or exemplify any property it does not actually exemplify. In this subsection I set out to analyze the objection from tautologous S-P discourse stemming from the first formulation. In the next subsection I proceed to analyze the objection from essentialism, which seems to be established by the second formulation.
The Bundle Theory of Particulars and Accidental Properties

The objection from tautologous S-P discourse questions the possibility of predicating attributes about particulars understood as bundles of properties. According to BT, in sentences like:

(a) This tomato is red.

(b) The fourth movement of Janáček’s Sinfonietta is in E flat.

the expression “this tomato” refers to a set of properties, one of which is the property of being red. The expression “the fourth movement of Janáček’s Sinfonietta” refers to a musical work that is constituted by a set of properties, one of which is the property of being in E flat. If so, these sentences say that the property of being red is one of the elements of a set to which it belongs and that the property of being in E flat is one of the elements of a set to which it belongs. Such sentences are tautologous and not informative, while the sentence (b) is informative. Thus, if this analysis were true, BT would be false.

The argument sketched above seems to be structured as follows:

Argument 3.1.1 against BT from tautologous S-P discourse
1. Particulars are bundles of properties [from BT].
2. Let \( a \) be a bundle of properties \( P, Q \) and \( S \) [1].
3. Bundles are sets [assumption].
4. So, \( a = \{P, Q, S\} \) [2, 3].
5. The “is” in S-P sentences amounts to “\( \in \)” [assumption].
6. So \( P(a) \equiv P \in a \), and so \( P(a) \equiv P \in \{P, Q, S\} \) [4, 5].
7. \( P \in \{\ldots P \ldots\} \) is a tautology.
8. \( P(a) \) is a tautology [6, 7].

First, we must note that the assumption (5) does not follow from the definition of BT. From BT we know that particulars are bundles of coexemplified properties and, I contend, we should analyse predication not in terms of set inclusion but rather in terms of coexemplification. Therefore, the “is” in sentences like (a) or (b) should not be equivalent to “in” but to “is coexemplified with” in such a way that (a) should be analyzed as “the property of being red is coexemplified with the property of being round and with the property of being a fruit”, if the tomato is constituted by these three properties. The sentence (b) should analogously be analyzed as “being in E flat is coexemplified with the property of being composed in 1926, the property (or relation) of being composed by Leoš Janáček, the property (or relation) of being first performed in Prague, etc.”

In general, if a particular object \( a \) is constituted by a bundle of three properties \( P, Q, \) and \( S \), then \( P(a) \) is not equivalent to \( P \in a \), where \( a = \{P, Q, S\} \), because \( P \in \{P, Q, S\} \) would be a tautology. I suggest that \( P(a) \) should be analyzed as equivalent to \( C(P, \{Q, S\}) \), where \( C \) is the primitive coexemplification relation provided by BT. \( C(P, \{Q, S\}) \) means that the property \( P \) is coexemplified with properties \( Q \) and \( S \). Sentences about coexemplification are not tautologous: the sentence “The property
of being red is coexemplified with the property of being round and the property of being a fruit” is equally informative as the sentence (a). Similarly, the sentence “Being in E flat is coexemplified with being composed in 1926, being composed by Leoš Janáček, being first performed in Prague, etc.” is not tautologous but is equally informative as the sentence (b).

Against this solution Bigaj (2012: 41-42) points out that in such an analysis the subject of the analyzed sentence is not preserved. If while uttering (a) we mean “the property of being red is coexemplified with the property of being round and the property of being a fruit”, then we express some information not about the tomato but about an entity \( tomato\text{-}minus\text{-}redness \). Also, note that if (a) is about \( tomato\text{-}minus\text{-}redness \), then if we had a similar sentence:

\[
(a^*) \text{ This tomato is round,}
\]

then \((a^*)\) would be about an entity \( tomato\text{-}minus\text{-}roundness \), while both (a) and \((a^*)\) seem to be about the same object — the tomato. Consequently, if the analysis of BT offered above were correct, then the expression “the tomato” would refer to different objects in these two sentences.

More generally, the objection says that if \( B \) is a bundle including properties \( \{P, Q, S\} \), then the subject of a sentence consisting in predicating \( P \) of \( B \) is \( B\text{-}minus\text{-}P \), and the subject of a sentence consisting in predicating \( Q \) of \( B \) is \( B\text{-}minus\text{-}Q \). Bigaj’s objection seems to be structured as follows:

Argument 3.1.2 against BT from variable subjects:
1. Sentences \( P(a) \) and \( Q(a) \) take the same subject \( a \) [assumption].
2. If \( P(a) \equiv C(P, \{Q, S\}) \) and \( Q(a) \equiv C(Q, \{P, S\}) \), then the subjects of these sentences are respectively \( \{Q, S\} \) and \( \{P, S\} \) [assumption from the formulation above].
3. \( \{Q, S\} \neq \{P, S\} \).
4. Sentences \( P(a) \) and \( Q(a) \) do not take the same subject \([2, 3]\).

In reply, it is enough to point out that there is no need to exclude the predicated property from the analyzed bundle. Although uttering “Redness is coexemplified with redness” would reinstantiate the tautology, the conjunction “Redness is coexemplified with redness, roundness, and being a fruit” escapes the original problem. Similarly, while uttering (b), we express a conjunction roughly as follows: “Being in E flat is coexemplified with being in E flat, being composed in 1926, being composed by Leoš Janáček, being first performed in Prague, etc.”

In general, I submit that any sentence of the form \( P(B) \) should be analyzed as a conjunction of statements about the coexemplification between the predicated property \( P \) and all (and not as Bigaj suggests, all-minus-\( P \)) properties in the bundle \( B \):

\[
P(B) := \bigwedge_{x \in B} C(P, x) \quad \text{(1)}
\]
For a = \{P, Q, S\} we get \( P(a) \equiv C(P, P) \land C(P, Q) \land C(P, S) \), and \( Q(a) \equiv C(Q, P) \land C(Q, Q) \land C(Q, S) \). Note some important and intuitive consequences of this proposal: the contents of these two sentences differ from each other in the same way as the contents of \( (a) \) and \( (a^*) \) do. The sentence about the time *Sinfonietta* was composed conveys different information than the sentence about its composer. Yet in all these cases the sentences are about the same thing — the tomato or the musical work.

If my argument is correct, then we have avoided the objections from tautologous S-P discourse and variable subjects in the language offered by BT at a very little cost.

### 3.2. Essentialism

A stronger formulation of the objection analyzed in the previous subsection is based not on the allegedly tautological character of S-P discourse in BT but on the necessity of that discourse.

There are numerous reasons to hold and defend the distinction between accidental and essential properties, even at the cost of accepting the ontological commitment to possible worlds. To establish such modal intuitions, Loux (2007: 100) introduces a small, light, red ball. Let us name the ball Sam and assume that Sam exemplifies four properties (and, for the sake of the argument, four properties only): let Sam be red \( (R) \), spherical \( (S) \), weigh 3 ounces \( (W) \), and be 2 inches in diameter \( (D) \). There seems to be strong intuition that Sam could have been slightly bigger or heavier without any harm to its identity, or that Sam could have been blue instead of red. These are the sentences that we called accidental in the first section of this paper.

Yet, as opposed to the case of colour and diameter, it seems that Sam could not have failed to be spherical, if it is to be a ball. This will serve as an example of a necessary property.

If Sam the ball is not enough to spark the reader’s modal intuition, let me introduce an example from natural kinds. Let us consider Eve the platypus. Eve has a duck bill \( (D) \), a beaver tail \( (T) \), otter feet \( (F) \), brown fur \( (B) \), she is 42 centimetres long \( (L) \), she lays eggs \( (E) \), and she is a mammal \( (M) \), giving \( e = \{D, T, F, B, L, E, M\} \).

Intuition has it that Eve could have been a couple of centimetres shorter or that her fur could have been of a slightly lighter shade, and I do not take that to mean that she could shrink over time or that she could turn grey while growing old. I take this intuition to mean that her history could have been different, for example in that she could have been born slightly smaller and thus grow to 39 centimetres instead of 42. These are examples of accidental properties. On the other hand, biologists would agree that Eve could not have been born oviparous nor could she have a trunk instead of her bill. Her being egg-laying and her having a bill constitute essential properties.

The last argument in favour of accepting the distinction between accidental and necessary properties that I want to put forward should be convincing even to readers who do not share the intuitions sketched above. The argument is that a formulation
of BT allowing for modalities would constitute a more general and thus more valuable theory. Accordingly, I submit, it is worthwhile taking the trouble to try and reconcile BT with accidental properties.

It has been argued that the attempt to postulate accidental properties in BT leads to certain difficulties. Van Cleve (1985: 122) points out that if bundles are understood as sets, then they inherit identity conditions from the said sets. This means that the identity of any bundle depends on their constituents just as the identity of any set depends on its elements. And so it seems that a bundle must have all the elements it actually has, for otherwise it would not be itself. But if so, then all the elements of a bundle are its elements necessarily, which directly opposes the intuitions sketched above. To be more precise, extensional sets have all their elements necessarily. Thus, if bundles are extensional sets, then indeed particular objects understood as bundles of properties cannot instantiate any of their properties accidentally.

The reasoning behind this objection looks analogous to argument 3.1.1, with necessity substituted for tautologousness. The answer to this challenge lies in modal realism.

Particular objects should be understood not as simple bundles of properties, but rather as bundles of bundles of properties, where a bundle constituting an object is a transworld entity, which means that it encompasses all possible worlds in which (pretheoretically) the object exists, and has as its elements the “simple” bundles of properties which constitute the particular modal counterparts in possible worlds. These inner-bundles constitute modal parts (analogously to temporal parts) of those objects we intuitively associated with first-order bundles according to the naïve interpretation of BT. The outer-bundles are extended not only along three (or four) dimensions but also across possible worlds.

Loux’s Sam, according to the account I am proposing, is not a simple bundle of properties. The bundle including the discussed properties of roundness, redness, etc. makes up just one element — a modal part — of a bigger, encompassing bundle. The bigger bundle includes as its elements bundles of properties from all possible worlds in which (we would pre-theoretically say that) there exists a counterpart of Sam. Eve the platypus turns out not to be a simple bundle of properties but rather a bundle extending over possible worlds whose elements are bundles of properties. If we were right in assuming that Eve could have had differently coloured fur (say, fox-red, \(R\), instead of platypus-brown, \(B\)) but could not have been oviparous, \(O\), instead of egg-laying, \(E\), then this means that in the bundle constituting Eve there is at least one bundle including the property \(R\) of having a fox-red coloured fur and that in the bundle constituting Eve there is no bundle including the property \(O\) of being oviparous:

\[
\mathcal{E} = \{\{D,T,F,B,L,E,M,…\},\{D,T,F,R,L,E,M,…\},…\}
\]

Such a simple modification saves the distinction between accidental and necessary properties. Eve’s colour is an accidental property insofar as there exists at least one sub-bundle of the transworld bundle constituting Eve that has a different colour as its
The Bundle Theory of Particulars and Accidental Properties

131

element. Eve’s being egg-laying is her necessary property insofar as there is no modal part of the outer-bundle that lacks the property of being egg-laying as its element. In other words, every modal sub-bundle includes the property of being egg-laying.

An important shortcoming of such a solution is that we need to revise our analysis of sentences including phrases like “it is possible”, “must”, etc. “Eve could have been \(F\)” no longer means that “There is a possible world in which Eve is \(F\)”.

3.3. Diachronic change

Yet another problem that is structurally similar to the argument from essentialism, also noted by van Cleve (1985: 122), is the problem of change. If particular objects were bundles of properties and if bundles were reducible to extensional sets, then no change could occur in the properties of the object, for no set can lose, gain, or change its elements. Thus, if we closely examine the case of a change of an object with respect to its properties, we see that a change while preserving the identity of the object is impossible. What really happens is that one object qua set of properties gets replaced by a new, numerically distinct object (where their difference is the image of the apparent “change”). For instance, if a particular object exemplifies properties \(P, Q,\) and \(S\) and ceases to exemplify \(P\) over time, this means that a new set \(\{Q, S\}\) replaces the old one \(\{P, Q, S\}\).

The solution to this objection is structurally similar to the reply to the argument from essentialism and can be discovered through the analysis of how objects exist in time. We must note that the problem of change is analogous to the problem of change in four-dimensionalism.

Four-dimensionalism (4D) is a doctrine stating that objects are temporally extended and perdure by having different temporal parts in different moments. 4D answers the problem of change by postulating that particular objects undergo changes in virtue of having qualitatively different temporal parts (Sider 1997: 3). Namely, an object undergoes a change because some of its temporal parts exemplify certain properties, while other temporal parts do not. In other words, an appearance or a disappearance of a certain property in the course of temporal parts of an objects constitutes a change of this four-dimensional object.

As Casullo (1988: 138) points out, it is true that if we “changed” one property in a bundle, then we would generate a new object. Yet we can avoid this conclusion if we postulate that particular objects are not bundles of properties but — analogously to our strategy in subsection 3.2 — bundles of higher order, four-dimensional objects extended not only spatially but also temporally. Elements of such temporally ex-
tended bundles would be what four-dimensionalists call temporal parts, and particular objects can undergo a change because they are partially constituted by different temporal sub-bundles exemplifying different properties.

If Eve had been brown \((B)\) at time \(t_0\) but was dyed fox-red \((R)\) sometime between \(t_0\) and \(t_1\), then this means that Eve is partially constituted by at least two temporal sub-bundles: the first one exists at \(t_0\) and includes \(B\), and the second one exists at \(t_1\) and includes \(R\) instead of \(B\):

\[
\mathcal{E} = \left\{ \frac{\{D, T, F, B, L, E, M, \ldots\}}{t_0} \right\}
\]

It is easy to see that combining the answers to the two above objections leads to identifying particular objects with five-dimensional bundles of bundles of properties, that is with objects that are spatially and temporally extended and exist across possible worlds.

The following summarizes the complete picture: suppose that Eve exists in worlds \(w_0\) and \(w_1\), at least at times \(t_0\) and \(t_1\). In \(w_0\) she has brown fur and is a mammal. She undergoes no change in \(w_0\). In \(w_1\) she is brown and is a mammal at \(t_0\), but her fur changes to red sometime before \(t_1\). Eve is necessarily a mammal because the property of being a mammal \((M)\) is included in every temporal sub-bundle of every modal bundle. Eve is accidentally brown because \(B\) is not included in every temporal sub-bundle of every modal bundle, namely, it is absent in \(w_1\) at \(t_1\). Eve’s fur changes colour because there is a temporal sub-bundle which includes the property of being brown (in \(w_1\) at \(t_0\)), and there is a sibling temporal sub-bundle (a temporal sub-bundle of the same modal bundle) which does not include the property of being brown (\(w_1\) at \(t_1\)):
The Bundle Theory of Particulars and Accidental Properties

Two worries could now be raised: first, that such an ontology is needlessly complicated and, second, that it is possible that the general strategy we applied in both cases — the strategy of escaping into higher-order bundles — would generate a regress.

In the first objection, we seem to be showing a healthy dose of skepticism in that we are wary of reducing common and simple things like chairs, trees, platypuses, and sinfoniettas to constructs very much detached from common experience, like bundles of bundles of bundles. But the opposition is premature. First, as I have already noted, while diachronic change seems to be universally accepted, some philosophers remain skeptical about modal situations and possible worlds. For them, the aim of this paper is purely theoretical and consists in proving the generality of BT in that it can accommodate accidental properties should someone want to. As a result, I presume, modal nominalists would limit the theory to the second step, where we postulated that bundles are temporally extended, but they would deny that bundles exist across possible worlds.

Second, a theory reducing a certain category of objects, say $x$s, to sets is by no means more categorially parsimonious than a theory reducing $x$s to sets of sets. It is not, because both theories carry ontological commitments to sets and their elements.

As for the second possible objection, it is enough to say, that the cases of diachronic and modal change are the only cases in need of such a reformulation of bundles, so that they can remain numerically identical even once their elements have changed. The fear of regress is thus premature, at least as long as someone provides a proof or at least an example of a purpose fourth-order bundles could serve, not to mention an infinite regress.

3.4. The Principle of Identity of Indiscernibles

The last objection against BT that is worth mentioning stems from Leibniz’s Principle of Identity of Indiscernibles. The objection runs as follows. If according to BT particulars are constituted by extensional sets and if two extensional sets are identical just in case they have the same elements, then if two particular objects exemplify the same properties, then they are not only qualitatively but also numerically identical. BT entails PII. But we know that PII is false, so BT must be false.

In the following paragraphs I give two possible answers to this objection. The first one requires only the modifications to BT we made in two previous subsections, while the second one will call for one further assumption.

In the two previous subsections we have abandoned the naïve formulation of BT and replaced it with a theory reducing particulars to either: (1) modal bundles of bundles of properties (which suffice to avoid the objection from essentialism but not to avoid the objection from change); or (2) temporal bundles of bundles of properties (which avoid the objection from change but do not avoid the objection from essen-
tialism — an option for modal nominalists); or (3) modal bundles of temporal bundles of bundles of properties (which avoid both objections). We have to note that in all three cases, qualitative identity of the lowest-order bundles does not lead to numerical identity of objects they constitute. Let us consider them one by one.

(1) In the first version of the theory, particulars are reduced to transworld bundles of their modal parts. But then we cannot speak of indiscernibles as soon as some objects in a possible world are qualitatively indiscernible, but only once all modal counterparts are pair-wise indiscernible (if a sub-bundle is a modal part, then let us define modal counterparts as all sibling sub-bundles of the bundle whose modal part constitutes this sub-bundle). So in version (1) numerical identity does not follow from qualitative identity between two modal parts but from necessary qualitative identity between two modal parts (that is, from qualitative identity of all the modal parts, which amounts to qualitative identity of the outer-bundles).

Strengthening the antecedent of PII into a necessary statement weakens the Principle of Identity into PII1. According to PII1, in order to find an object that is indiscernible from *Sinfonietta*, we need to find another musical work not of identical structure (assuming that a musical work’s structure is constituted by its properties) but a work that in every possible world has a structure identical to *Sinfonietta’s* in that world. I believe this weakens PII beyond any practical threat.

Version (2) weakens PII in a similar way. PII2 is a principle according to which only those particulars are indiscernible which exemplify the same properties at the same times. In other words, it is their complete histories that need to be indiscernible.

Version (3) weakens PII even further. In a reasoning analogous to the two above, we see that we would find indiscernibles only if we found two particular objects with indiscernible complete histories in all possible worlds. In the present discussion, the antecedent of PII3 is reduced to an extreme case in which *Sinfonietta* would be qualitatively identical only with a musical work which necessarily shares its structure and history. Since diachronic change in musical works is a disputable thing, consider Eve: a platypus would be indiscernible from Eve if it existed in the same possible worlds as Eve and if in each of these worlds it exemplified exactly the same properties at exactly the same times (that is, underwent the same changes) as Eve in that world.

We need to note that versions (1)-(3) do not trivialize PII completely. Even in the last case it is still possible to construct a scenario in which two numerically different objects would be (modally and temporally) indiscernible. If we needed to dissolve PII trivially and completely, I would suggest looking into the Trope Theory. To prevent the entailment from BT to PII, it is enough to reject the theory of properties based on universals. As is easy to show, PII turns out to be trivially false once we assume both BT and TT. From TT we know that there exist intrinsically indiscernible tropes that nevertheless remain numerically distinct. And thus we can have two bundles consisting of qualitatively identical properties. If those properties are tropes, they have to be numerically distinct, and so BT and TT falsify PII.
CONCLUSIONS

If my argument is correct, I have shown a way to reconcile the Bundle Theory of Particulars with informative subject-predicate discourse, with the distinction between accidental and necessary properties, and with a notion of change over time. I have also explained why the offered solutions weaken (but not trivialize) the Principle of Identity of Indiscernibles.

The proposed solutions consisted in analyzing subject-predicate discourse in terms of coexemplification of properties rather than set-inclusion and in postulating that ordinary particulars are not bundles of properties but bundles of higher-order: either bundles of temporal parts (that are bundles of properties), which allows us to escape the problem of impossibility of change over time; or bundles of modal parts, which enables us to refute the argument from essentialism; or bundles of modal parts that are bundles of temporal parts that are bundles of tropes — which clears up both difficulties.

REFERENCES