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EXPLORING THE LAGRANGIAN SCHEMA IN MEETING EXPECTATIONS AS AN ALTERNATIVE TO THE NEWTONIAN SCHEMA

Abstract

Lee Smolin believes that current physics and modern cosmology are in crisis because of the application of “the Newtonian schema,” according to which the universe is like a computer that receives initial conditions, and the governing laws then generate its subsequent states. According to him, it is the application of the Newtonian schema that has led to common but false beliefs, such as the belief in the unreality of time or the belief in the reality of the multiverse. It is necessary, Smolin concludes, to abandon this schema in order to overcome this crisis, but he proposes no alternative methodology. The only available alternative has been proposed by Ken Wharton, who suggests replacing the Newtonian schema with “the Lagrangian schema,” which, unlike the Newtonian schema, offers a holistic and global point of view. According to the Lagrangian schema, the system under study or the world as a whole is examined in an all-at-once manner, rather than as the time evolution of a timeless part. This article aims to investigate Smolin’s criticisms of the Newtonian schema and Wharton’s proposal for replacing it with the Lagrangian schema. We intend to show that although the Lagrangian schema does not suffer from some of the problems of the Newtonian schema, it still faces some similar challenges. For example, even if one uses the Lagrangian schema, time can still be unreal, and the multiverse can exist. Thus, Smolin is likely to find the Lagrangian schema problematic.

Keywords: Newtonian schema, Lagrangian schema, cosmology, time, multiverse, Lee Smolin, Ken Wharton

Lee Smolin, a prominent physicist and one of the founders of loop quantum gravity, believes that modern cosmology is in crisis and is incapable of answering

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some of our most important cosmological questions. This crisis, he claims, is because of the application of a special methodology in physics, which he calls “the Newtonian schema” (Smolin 2009) or “the Newtonian paradigm” (Smolin 2015, Unger and Smolin 2015). According to the Newtonian schema, the world is just like a computer that receives inputs, and the governing laws then generate outputs. The inputs of this cosmic computer are the initial state, or – more familiarly for physicists – the initial conditions of the universe, and the outputs are its subsequent states at later times. According to Smolin, it is the application of the Newtonian schema that has led to common but false beliefs such as the belief in the unreality of time or the belief in the reality of the multiverse. So, he considers it necessary to abandon this schema to overcome the crisis.

However, Smolin proposes no alternative methodology. As far as we know, the only available alternative has been proposed by Ken Wharton, a prominent quantum physicist, who suggests replacing the Newtonian schema with what he calls “the Lagrangian schema” (Wharton 2013a, Wharton 2015b). Unlike the Newtonian schema, the Lagrangian schema offers a holistic and global point of view, according to which systems are examined in an all-at-once manner, not as the time evolution of a timeless part. The Lagrangian schema refutes the “cosmic computer” understanding of the universe and renders the computational philosophy behind it *locally* true. This article aims to examine Smolin’s criticisms of the Newtonian schema and Wharton’s proposal for replacing the Newtonian schema with the Lagrangian schema. We intend to show that although the Lagrangian schema does not suffer from some of the problems of the Newtonian schema, it still faces some similar challenges.¹

The organization of this article is as follows: Sections 1 to 4 are devoted to a review of Smolin’s criticisms of modern cosmology and the Newtonian schema. In due course, we discuss some criticism. Section 5 is devoted to Wharton’s important criticism of the Newtonian schema, which he considers to be the new embodiment of anthropocentrism. Section 6 is devoted to the introduction of the Lagrangian schema as a potential alternative to the Newtonian schema. In Section 7, we discuss an objection that Wharton’s proposal may be faced with. In Section 8, we evaluate the advantages of the Lagrangian schema over the Newtonian schema and attempt to determine whether it meets Smolin’s expectations. Finally, we conclude in Section 9.

¹See, also, (Maghsoudi and Taheri Khorramabadi 2024).

1. A CRISIS IN PHYSICS AND THE NEED FOR A REVISION IN THE CONCEPT OF “LAW”

Smolin believes that there is a crisis in physics regarding the failure to provide a scientific explanation for some facts of the standard models of particle physics and cosmology, as well as the failure to integrate the two. He states that:

The crisis is due to our inability to go deeper than these models to a further unification of physics or to explain the features of the models themselves. They reveal a universe that on rational or aesthetic grounds appears preposterous, and each has a long list of parameters which must be tuned very finely to agree with experiment. Many ideas have been proposed to explain why these parameters have the values they have; none has definitely succeeded. (Unger and Smolin 2015: 354)

He points out that:

There is no crisis if our attention is just restricted to the data itself. . . . We can model the observations using standard general relativity and quantum field theory. . . . The crisis is rather in attempts to go beyond modeling, *to explain* the data. . . . We confront crisis when we expand our ambitions from describing *the part of the universe we can see* to having a theory of *the whole universe*. (Unger and Smolin 2015: 360, our emphasis)

It is clear that what he means by “crisis” is *a crisis in explanation*. Smolin believes that to overcome this *crisis* it is necessary to revise the concepts of “scientific explanation” and “natural law.”²

His starting point is a question: How has our current understanding of natural laws been affected by our ways of experimentation? We do not study the whole world *in the laboratory*. Rather, we examine a small subsystem of the world that can be considered isolated if we ignore the influences of the tools we use for measurement and the interventions we make. To determine the “natural law” that governs the system at study, we assume that we can repeat the experiment whenever and wherever we want while the components of that system are fixed and the configuration of those components is different. We call the regularity that remains fixed in all these situations a “natural law,” while attributing the differences in results to differences in “initial conditions.”

²See (Smolin 2009, 2015) and (Unger and Smolin 2015). See, also, (Burton H. (ed.) 2021), which is an interview with Smolin, and (Smolin 2013b), which is written for the public audience. Also, see (Gaspar and Tambor 2023) for a discussion about the problems concerning the definition of “law” in cosmology.

It is this type of investigation that makes it possible to sharply distinguish between the concepts of “natural law” and “initial conditions.” This distinction leads to one of the most widely used concepts in physics, namely the concept of “state space,” which is the geometric space of all possible states of a system. From this point of view, the laws of motion determine the evolution of the system from a point in state space, known as initial conditions, to its subsequent points over time. Smolin calls this approach “the Newtonian schema,” “the Newtonian paradigm,” or “the Newtonian approach to scientific explanation,” which is based on:

- defining a subsystem of the world and ignoring its interaction with its surroundings,
- determining the space of possible states of that system (state space), and
- determining how that system evolves over time.

Together, these three steps form the “standard methodology of physics” (Unger and Smolin 2015: 373).

The first step requires the observer to be located outside the system, along with a clock that measures the time and is also outside the system.³ This observer can influence the system through measurement, but the observer’s influence is assumed to be negligible or is absent in the evolution equation, like the situation in collapse theories of quantum systems. The second and the third steps formulate the kinematics and dynamics, respectively. The structures employed in formulating kinematics are the configuration space or phase space in Newtonian mechanics, and Hilbert space in quantum mechanics. This dynamic formulation utilizes a foliation of the space, complemented by a family of paths, each point of the space being traversed by precisely one such path.

Furthermore, the first step requires the results obtained from the Newtonian schema to be only *approximately* true. The Newtonian schema applies to isolated systems, but no system in the real world is isolated because real systems interact with each other: “at worst, it is physically impossible to shield a system from the influence of gravitational waves coming from the outside” (Smolin 2015: 92).

³Smolin points out that there are specific classical systems whose time parameters can be given by the internal time of the system (see Smolin 2015: 91 ft. 8). A simple example of such systems is the parameterization of the evolution of a single particle in terms of its proper time, which immediately makes our formulation generally covariant. However, such systems are exceptions.

When we consider a system in isolation, we ignore interactions. Therefore, the Newtonian schema does not provide a completely accurate picture of the world but an approximation that, of course, is experimentally successful.⁴

The Newtonian schema has three important features: First, this schema, as mentioned above, is based on the sharp distinction between kinematics and dynamics. Second, both kinematics and dynamics are formulated in a “timeless” (Smolin 2015: 91) manner, i.e., in such a way that the structures employed do not change in time:

States . . . correspond to possible preparations or complete measurements, as done by an observer using instruments external to the system. Evolution is defined with respect to a clock external to the system. To proceed one makes a series of measurements at times t_1, t_2, \dots to determine the state at each time. This resulted in a record, $\{p(t_1), p(t_2), \dots\}$. This record, once made, is static, it doesn’t change in time. It is therefore entirely appropriate to represent the record by a curve $\gamma(t)$, which coincides with the entries in the record at the stated times. This is a mathematical object, which is also unchanging in time. (Smolin 2015: 92)

Third, the Newtonian schema, contrary to its name, is not limited to the boundaries of Newtonian mechanics, but its scope of application includes classical mechanics, quantum mechanics, general relativity, quantum field theory, quantum gravity, and some computational models.⁵

2. THE COSMOLOGICAL FALLACY

Smolin considers the Newtonian schema as the basis upon which some have claimed that time is unreal. According to him, time in the Newtonian schema is nothing but a parameter on a path in the state space; it is not an inherent part of nature:

Time in the Newtonian schema is a parameter used to label points on a trajectory describing the system evolving in [state] space. When the system is small and isolated,

⁴Physicists are well informed of this fact and, as Smolin has noted, they consider our best scientific theories and models, such as quantum chromodynamics, general relativity, and the standard model of particle physics, as *effective field theories* and models derived from them; see (Smolin 2015) and (Unger and Smolin 2015: 375).

⁵See (Smolin 2015), (Unger and Smolin 2015: 359), and (Wharton 2015b). Also, Smolin points out that the Newtonian schema underlies the standard framework of computer sciences and the mathematical models commonly used in the biological and social sciences (Unger and Smolin 2015: 373).

this time parameter refers to the reading of a clock on the wall of the observer's laboratory, which is not a property of the system. When we try to apply this notion to the universe as a whole, the time parameter must disappear. Some have attempted to argue that this means that time itself does not exist at a cosmological scale, but that is the wrong conclusion. What disappears is not time, but the clock outside of the system – which would be an absurd object since the system is the whole universe. (Smolin 2009: 24)

According to Smolin, the notion that “time is unreal” cannot be reliably deduced from the efficiency of the Newtonian schema because such an argument is fallacious:

- The Newtonian schema is experimentally successful.
- According to the Newtonian schema, time is not real at the fundamental level of reality.
- Therefore, time is not real at the fundamental level of reality.

When we observe the motion of, say, a moving particle, we record measurements of its positions over time. The obtained results can be visualized in the state space as a curve whose points represent the states of the moving particle. This diagram is timeless, in that it is a representation of recorded positions that will never change. This does not mean that real motion is also timeless, “[n]or does it imply that behind the real evolution in time of the real world there exists a complete correspondence to a timeless mathematical object. To posit this further relation is a pure metaphysical fantasy, which is not implied by anything in the science” (Smolin 2009: 24).⁶ Just as we are not allowed to conclude that the world is a mathematical entity from the fact that there is a mathematical description of the world, we are not allowed to conclude that the world is timeless from the fact that the said mathematical description is timeless. Smolin calls the wrong argumentation “the cosmological fallacy,” which, according to him, has its roots in the point being missed that the Newtonian schema is *not* applicable to the world as a whole:

⁶Smolin calls this thesis that there is a mathematical object isomorphic to the history of the universe the “Pythagorean dogma” and asserts that “Nor can any mathematical object serve as a complete mirror of the universe or its history, in the sense that every property of the universe is mapped to a property of that mathematical object” (Unger and Smolin 2015: 358).

To ignore this and attempt to scale up the Newtonian [schema] to the universe as a whole is to commit the cosmological fallacy. An important example of the cosmological fallacy and a false deduction from it is the claim that the right interpretation of the equations of classical or quantum cosmology is that the universe is timeless. Classical and quantum models of cosmology appear timeless because they arise from applying to a system with no external clock a method and formalisms whose empirical context requires an external clock. To deduce from the formalism of classical or quantum general relativity that the universe is timeless is fallacious. (Smolin 2015: 92)

Smolin thinks that due to the many problems that classical and quantum cosmology are facing, and because these two are largely based on the Newtonian schema, the true metaphysical picture of the world as a whole cannot be based on the Newtonian schema. His main idea is that there is no empirical evidence to support the sharp distinction between the concepts of law and of initial conditions *in cosmology*.

A system can be prepared with various configurations for repetitious experiments or observations, but the universe cannot be set up with various configurations: the world is unique. So, “it is not clear what meaning a *general* law has in this context” (Unger and Smolin 2015: 375, our emphasis). If we cannot replay the tape of the history of the system at study more than once with different initial conditions, then we will face the problem of “degeneracy,” according to which the same set of experimental data “can be explained by different choices of laws and initial conditions that cannot be resolved by doing experiments with more cases” (Unger and Smolin 2015: 375).⁷ Smolin points out that this problem reduces the theory’s predictive and postdictive power.

“If there is just one universe, there is no reason for a separation into laws and initial conditions, as we want a law to explain just the one history of the one universe” (Smolin 2009: 24). The same is true about the state space of the world: Smolin argues that the concept of “states of the world that exist in its state space but are never realized” is meaningless because the world is realized only once.⁸

⁷The problem of *degeneracy* should not be conflated with that of *underdetermination*. While the latter is due to the multiple-confirmation of different theories based on a single set of empirical data, the former is due to the multiple-articulation of a single theory based on a set of empirical data when the differences between those articulations are in what is considered as the law and what is considered as the initial condition.

⁸In a similar vein, he considers the concept of the “quantum state of the universe” that is used in quantum cosmology a “fiction” (Smolin 2009: 24).

Such concepts are useful when applied to subsystems of the world, not the world as a whole.

It may be objected that the opponent's argument is not that "the Newtonian schema is experimentally successful; according to the Newtonian schema, time is not real at the fundamental level of reality. Therefore, time is not real at the fundamental level of reality." This argument, however, is not deductive *at all*; it is an induction, abduction, or inference to the best explanation that can be articulated, for example, as follows:

- The Newtonian schema has been experimentally successful when applied to most, if not all, subsystems of the world.
- Therefore, it is very likely that the Newtonian schema is also successfully applicable to the world as a whole.
- According to the Newtonian schema, time is not fundamentally real.
- Therefore, it is very likely the case that time is not fundamentally real.

Smolin replies that the degree of success in applying the Newtonian schema is *context dependent*. This, in turn, is because ignoring the influences of the environment or considering the system isolated, which is what the application of the Newtonian schema requires, is highly dependent on the context in which the subsystem is defined.

This dependence on the context of a subsystem of the universe blocks any broad metaphysical conclusions being drawn from the success of the Newtonian paradigm, because any such deduction would require the application of the paradigm to the whole universe. (Smolin 2015: 92)

All successful applications of the Newtonian schema have so far been about subsystems of the world, not the world as a whole. So, the second line of the above argument is false.

It may be said that this argument is based on an *analogy* between subsystems of the universe and the universe itself, of the kind that has generally been useful in physics. Physicists often use analogy when studying systems. For example, an electric circuit containing an electromotive force e , an electrical capacitance C_E , an electrical resistance r_E , and an inductance L connected in series is similar to a system containing a driving force f_M acting on a particle of mass m attached

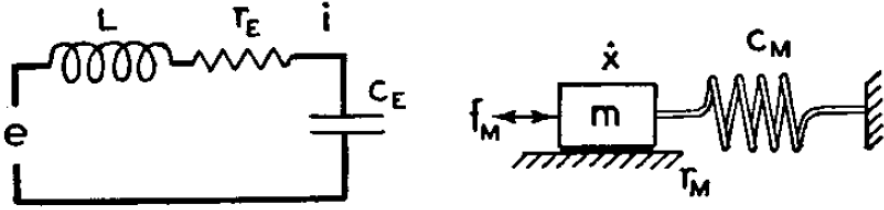


Figure 1. Right: The mechanical system. Left: The electromagnetic system (Olson 1943: 25).

to a spring of compliance C_M sliding on a frictional plate of resistance r_M (see Figure 1).

This example demonstrates the possibility of using analogies between mechanical and electromagnetic systems to learn about a system by studying another system.⁹ Cosmologists also use analogies extensively to learn about the universe as a whole from the subsystems of the universe.¹⁰ However, Smolin considers this use of analogy to be invalid *for explanatory purposes*. He believes that this kind of analogy which has no direct empirical support leads to the mentioned failures of explanation. Despite its great empirical achievements, the Newtonian schema is not successful *in explaining* the universe as a whole.¹¹

⁹Harry F. Olson (1943) has discussed many examples of such analogies in physics.

¹⁰Valerio Faraoni (2023) presents many examples of analogies used in cosmology, in each case explaining what we learn about the cosmos from the subsystems around us. Regarding the critical role that analogy plays, he insists that “[w]hen intuition fails, an analogy can guide us” (Faraoni 2023: viii).

¹¹Gaspar and Tambor (2023) address an important explanatory difference between cosmology and other physical sciences. They point out that “Cosmology, rather than explaining laws for the entire Universe, concerns itself with constructing a model, i.e., a representation of the Universe, to provide a consistent and relatively complete description of what our Universe is like” (Gaspar and Tambor 2023: 18). There is something special about the model here: “A cosmological model is not a model in the same sense as a planetary one because it does not represent any specific physical system that functions as a part of a greater whole; instead, it is the model of everything that physically exists. In cognitive terms, a cosmological model is bound with the universe, which humanity cannot learn about other than through a model. Furthermore, a cosmological model represents the universe more than the theory it was built on” (Gaspar and Tambor 2023: 19).

3. A MISUNDERSTANDING CALLED “MULTIVERSE”

Smolin moves on and concludes that this unjustified adherence to the Newtonian schema *outside of its domain of validity*¹² leads to the multiverse hypothesis. The multiverse hypothesis, he thinks, is a false hypothesis

according to which our universe is just one of a vast or infinite collection of other universes, within which the properties we have failed to explain – like the parameters of the standard models of [particle] physics and cosmology – are distributed randomly. This surrender of the hope for sufficient reason – the hope to satisfy our curiosity as to the root of things – is nothing but an indication that a philosophy wrongly assumed to be an essential part of science has failed. (Unger and Smolin 2015: 357)¹³

From Smolin’s point of view, what has happened is as follows: we have applied the Newtonian schema outside of its domain of validity, i.e., concerning the world as a whole, while this application is already doomed to failure. When faced with the challenges of this wrong decision, instead of going back and identifying and fixing the root of the problem, we wrongfully decided to take a step forward and assume that our universe itself is a subsystem of a larger system, i.e., the multiverse:

We get to do physics as we have been trained to, but this is a trap because to do this we must employ structures that have no *operational significance*. Better, in our view, to regard the Newtonian schema as inapplicable to cosmology, and to look for another notion of law that can make sense when applied to our entire, but single, universe. (Smolin 2009: 24; our emphasis)

This unwarranted and invalid expansion of the scope of application of the Newtonian schema from the subsystems of the universe to the world as a whole reduces rather than increases the experimental adequacy of the Newtonian schema. Smolin calls this the “cosmological dilemma.”

The dilemma we are facing is as follows: we are provided with a seemingly general law that is empirically proven to work successfully in many, if not all, cases where we examine a subsystem of the world. However, what is meant by “successfully” should always be considered *approximately*, that is, the result was

¹²By “the domain of validity,” Smolin means where its application is consistent with empirical practice, i.e., where the triple steps of the instruction that the Newtonian schema dictates (see Section 1) can be implemented. He points out that the application of the Newtonian schema where its application is valid “underlies the dramatic success of physics since Galileo, Kepler and Newton” (Smolin 2015: 91).

¹³See also (Smolin 2007, 2013b), where he has raised various criticisms against the multiverse hypothesis.

empirically successful to a certain degree of accuracy that was included in our investigation. In an approximation, some interactions between the subsystem and its surroundings are omitted. So, to improve the result, these interactions should be included step by step. In other words, “[t]o make the application of the law more exact, one can seek to expand the subsystem to include interactions with and dynamics of an increasingly larger set of degrees of freedom” (Unger and Smolin 2015: 376). Thus, the result becomes more and more accurate; however, the question is whether there is an upper limit to this process, and this is where the cosmological dilemma appears.

If we assume that there is no upper limit to the increase in accuracy and that the result can be constantly made more accurate, i.e., if we accept that it is possible to increase the accuracy by adding step by step the influence of all the interactions in the world, then we will finally reach a stage where we find ourselves surveying the entire world. “[A]t that point there is only one case and one run of each measurement so the operational context which defined the notion of a general law no longer applies” (Unger and Smolin 2015: 376). In this situation, as argued earlier, the *general law* has lost its meaning and cannot be distinguished from the initial conditions. Therefore, what we were trying to increase the accuracy of disappears during the process of increasing the accuracy. Another way is to accept that our *general law* will not become more precise than that. If so, we are no longer justified in applying our *general law* to the world as a whole. Facing this dilemma means the failure of the standard scientific methodology because any of the forward paths we choose will lead to failure.

Furthermore, Smolin’s opposition to the multiverse hypothesis is partly because he believes that this hypothesis is not empirically falsifiable, therefore it is not scientific. He is unhappy that physicists use the concept of the multiverse so widely. However, his belief has significant opponents. Martin Rees, an eminent cosmologist, believes that the multiverse hypothesis is genuinely scientific and falsifiable. He points out that if we define the “universe” as “everything that exists,” the multiverse would be meaningless; however, if we define it as “the domain of space-time that encompasses everything that astronomers can observe” (Rees 2003: 210), the multiverse is quite possible and probably exists. To explain the falsifiability of the multiverse hypothesis, he points to a scenario suggested by Smolin himself.¹⁴ According to that scenario, there is a kind of inheritance

¹⁴For more details, see (Smolin 1992, 1997, Smolin 2013a).

among the universes of a multiverse such that new universes are spawned inside the black holes of the parent universe.

If Smolin were right, universes that produce many black holes would have a reproductive advantage that would be passed on to the next generation. Our universe, if an outcome of this process, should therefore be near-optimum in its propensity to make black holes, in the sense that any slight tweaking of the laws and constants would render black hole formation less likely. (I personally think Smolin's prediction is unlikely to be borne out, but he deserves our thanks for presenting an example that illustrates how a multiverse theory can in principle be vulnerable to disproof.) (Rees 2003: 219)

4. AN EVEN DEEPER CRISIS AND THE NEED FOR A NEW SCHEMA

In addition to the two unpleasant theses of the unreality of time and the reality of the multiverse, both of which are products of the invalid application of the Newtonian schema, there is also another important problem: according to Smolin, the Newtonian schema is incapable of trying to answer three important cosmological questions:¹⁵

- (1) Why do these particular dynamic laws, and not others, govern the universe?
- (2) Why did the universe begin with these particular initial conditions and not others?
- (3) Why has the universe not yet reached thermodynamic equilibrium after about 14 billion years?

This is because dynamical laws and initial conditions are the *inputs* of the Newtonian schema, not its *outputs*.

It can be said that the arbitrariness inherent in the choice of laws and initial conditions implies that the Newtonian schema leaves the first two questions unanswered. The third question can be considered a different articulation of the second one. In the framework of the Newtonian schema, this question can be answered as follows: the universe began with initial conditions on the basis

¹⁵Smolin calls the first two questions together “the landscape problem” (see Smolin 1997). For a review of the various challenges facing modern cosmology, see (Gaspar and Tambor 2023).

of which the initial entropy of the universe was so low that, after the passage of about 14 billion years, its increase under the second law of thermodynamics has not yet reached the maximum.¹⁶ So, the third question now is why did the world begin with such an improbable state? Smolin considers the most fundamental challenges facing current cosmology to be these three questions, which he considers rooted in the invalid application of the Newtonian schema to the universe as a whole.

What could be the alternative to the Newtonian schema? Smolin has not made a specific proposal, only suggesting that the solution lies in accepting “temporal naturalism.”¹⁷ He points out that there is no complete answer to the question “What would physics be like without a sharp distinction between the law and initial conditions?” However, he is certain about three points. First, the multiverse hypothesis should be abandoned, and the concept of “law” in cosmology should apply to only one universe, i.e., our universe. Second, time is fundamentally real. Third, the law in cosmology cannot be outside of time; rather, it should be inside time and prone to change over time: “a law where the distinction between a one-time narration of the history of the one universe and the statement of principles governing that history weakens” (Smolin 2009: 26).

At the same time, Smolin has mentioned another point that could be a hint for overcoming the crisis of modern physics and cosmology: there is a deeper crisis in mechanical philosophy, or its reincarnated version, i.e., computational philosophy.¹⁸

The heady idea that all that exists is natural, physical stuff is more plausible now than ever, due partly to progress of physics and digital technologies, but even more to the triumph of reductionist strategies in biology and medicine. Yet it is in crisis because of an embrace of the old metaphor that the world is a machine. In its modern incarnation

¹⁶Smolin points out that “there are parts of the universe that are in thermal equilibrium, like the microwave background, but there are big pieces of the universe which are not in thermal equilibrium” (quoted from (Burton H. (ed.) 2021)).

¹⁷Smolin is not against naturalism, he only criticizes its conventional version. From his point of view, the solution to all these crises is to adopt a new kind of naturalism committed to the reality of time and the evolution of laws. According to this type of naturalism, there is no timeless and universal law. See (Smolin 2015) and (Unger and Smolin 2015: 357–358, 362–363). Also, he believes in the principle that philosophers call “the principle of the causal closure of the physical” and takes it as one of the pillars of the doctrine of the evolution of laws – the other is the reality of time. See, also, (Smolin 2001, chap. 1).

¹⁸He noted elsewhere (Smolin 2012) that this crisis has, in turn, led to a crisis in the program of strong artificial intelligence and the philosophy of mind.

the mechanical philosophy becomes the computational philosophy that everything, including us, are, or are isomorphic to, digital computers carrying out fixed algorithms. (Unger and Smolin 2015: 356)

The Newtonian schema takes the world as a computational mechanism that receives initial states as inputs and generates future states as outputs of that computational mechanism.

According to Smolin, the alleged crisis in physics can be traced back to the fact that conventional naturalism wrongfully adheres to the doctrine that the world is a kind of machine, or equivalently, that the world is a kind of computer. Wharton (Wharton 2013a, 2015b) has also addressed this issue.

The widely accepted Newtonian schema, although once considered indisputable, now appears more open to revision than ever before. Wharton (2013a, 2015b) discusses in detail the problems of the Newtonian schema in explaining quantum phenomena. Also, the Newtonian schema is tightly bonded with an approach regarding the laws of nature known as “dynamic production,” according to which the laws governing nature are differential equations with respect to time, which by receiving the state of the system at a certain moment as an input produce its subsequent states. As Emily Adlam (2022) and E. K. Chen and Sheldon Goldstein (2022) point out, (1) dynamic production conflates the concept of determinism with predictability, while the former is often considered metaphysical and the latter epistemological. Furthermore, due to the failure to provide appropriate explanations for quantum nonlocality and delayed choice experiments, (2) dynamic production does not provide us with an appropriate metaphysical picture of the quantum world. Moreover, (3) it is not clear how dynamic production can be properly applied to spacetimes that do not contain a *first* instant of time, such as spacetimes that contain initial singularities, or spacetimes that contain closed time-like curves, such as Gödel’s spacetime. These criticisms, with minor revisions, can also be leveled against the Newtonian schema.

5. ANTHROPOCENTRISM

But how could such a widespread and seemingly innocuous attitude be wrong? Wharton (2015b) provides us with an answer that simultaneously clarifies what is wrong with the Newtonian schema and explains why this doctrine is widespread among physicists and philosophers. He points out that the Newtonian schema

is anthropocentric, that is, it envisions the physical world as we humans solve physics problems.

When examined critically, [this] assumption is exactly the sort of anthropocentric argument that physicists usually shy away from. It's basically the assumption that the way we humans solve physics problems must be the way the universe actually operates. (Wharton 2015b: 178)

From an anthropocentric viewpoint, we human beings are at the center of the universe. When interpreted epistemologically, the claim is that our scientific knowledge, i.e., our descriptions and explanations of natural phenomena, is formed based on the assumption that we are at the center of the universe.¹⁹ When interpreted ontologically, the claim is that the world is exactly the way we are used to thinking about it. While the former seems innocuous, the latter is the kind of human superiority that Wharton opposes. Anthropocentrism once led to the geocentric model of the solar system, which was the predominant yet incorrect description of the known universe for almost two thousand years. Today, anthropocentrism still causes problems. From this anthropocentric viewpoint, the universe is a computer because at least since Newton we have thought of physical theories as frameworks into which we feed initial conditions, and the machinery of laws then generates outputs.

The Newtonian schema is aligned with our everyday experiences, intuitions, and common sense. However, studying the fundamental level of reality is the task of physical theories, not common sense.²⁰ Although this claim may have opponents and critics, it has been widely accepted, at least in the current atmosphere of physical sciences. So, if we accept such a view, it is not surprising to find out that the Newtonian schema is not the appropriate metaphysical framework for physical theories.

¹⁹It should be noted that accepting the ability to acquire knowledge about the world does not require the belief that we humans are at the center of the world. It is not the case that denying skepticism requires accepting anthropocentrism. The claim is simply that the Newtonian schema is constructed in such a way that it is *as if* we humans were at the center of the universe.

²⁰The point is that common sense alone is not reliable enough to discover the fundamental level of reality, and it needs add-ons. We do not intend to deny the possibility of knowledge of the world.

6. THE LAGRANGIAN SCHEMA

What are we left with if we abandon the Newtonian schema? What does physics look like without the Newtonian schema? As mentioned, Smolin has not provided us with a specific answer. However, Wharton (2013a, 2015b) proposes replacing the Newtonian schema with the Lagrangian schema,²¹ according to which the equations of motion are obtained from the minimization of the *action*, which is the time integral of a scalar called the *Lagrangian*. For example, the Lagrangian is nothing but the difference between the kinetic and potential energies within Newtonian mechanics. Moreover, the path along which the system moves from one point of the state space to another, among all the possible paths the system may take (given the constraints on the paths), is the path that minimizes the time integral of the difference between the kinetic and potential energies. In general, the path followed is the path that minimizes the time integral of the Lagrangian.

A well-known example of the application of minimal principles is the use of Fermat's principle of least time, according to which light rays travel by a path that requires the least time. Fermat's principle justifies the laws of both reflection and refraction *simultaneously*.²² Given Fermat's principle, it is possible to explain why a light ray follows a broken line when passing, say, from air into water (Figure 2).

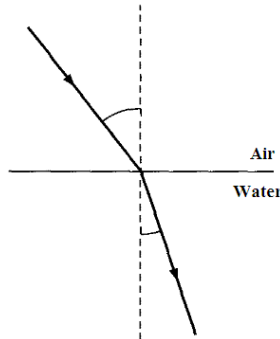


Figure 2. The path of a light ray passing from air into water.

²¹See also (Wharton 2013b, 2015a, 2016) and (Adlam 2018, 2022).

²²The laws of light reflection were explained by Heron of Alexandria, who lived about 100 A.D., by resorting to a minimal principle: light takes the shortest path. But this principle is unable to explain the laws of refraction. For an interesting review of the application of minimal principles in geometric optics, see (Schiffer and Bowden 1984, chap. 3).

Within a homogeneous medium, the least-time path between two points is the straight-line segment connecting those two points. However, since light travels faster in air than in water, when passing from air into water the least-time path consists of two straight-line segments, one in the air and one in the water, in such a way that the time duration of the path traveled by the light is minimized in conformity with the rules of the calculus of variations.²³

But how does the light *know* at the starting point that somewhere in its path it will enter water so that it can adjust the initial angle in such a way that at the end it turns out that the path the light traveled is the least-time path? More generally, how do physical systems initially *know* how to start their evolution so that their action turns out to be minimized when they conclude? As Wharton points out, unlike the Newtonian schema, the behavior of systems is not explained by an “algorithm-like chain of cause-and-effect” within the Lagrangian schema, “but rather because it’s globally more efficient” (Wharton 2015b: 181).

To summarize the Lagrangian Schema, one sets up a (reversible) two-way map between physical events and mathematical parameters, partially constrains those parameters on some spacetime boundary at both the beginning and the end, and then uses a *global* rule to find the values of the unconstrained parameters. These calculated parameters can then be mapped back to physical reality. (Wharton 2015b: 182; our emphasis).

The Lagrangian schema uses a *global* or *holistic* point of view. Yoichiro Nambu, a prominent physicist and 2008 Nobel Prize winner, has also mentioned a similar point, this time regarding the Lagrangian formulation of the quantum theory:

Feynman’s theory [the Lagrangian formulation of quantum theory] though in most respects equivalent to ordinary quantum mechanics, has revealed us a much larger freedom and variety in the ways of attacking individual problems. Thus, for example, one can eliminate some of dynamical variables (“field”) and replace them by an equivalent action at a distance. After such a procedure, however, the ensuing equivalent dynamical system cannot be fitted into the frame of the ordinary Hamiltonian formalism,²⁴ but has to be treated as a “non-local” system, i.e., a system having integral equations for its equations of motion. In view of this, it is desirable to extend our investigations to include systems with higher derivatives as well as non-local systems. (Nambu 1952: 2)

²³See (Baez and Wise 2019), (Thornton and Marion 2004) (introductory), and (Lanczos 1952) (advanced).

²⁴More on this soon. Also, see ft. 23.

Therefore, the Lagrangian schema *metaphysically* differs from the Newtonian schema: while the Lagrangian schema enjoys a global and holistic viewpoint, the Newtonian schema benefits from a local and reductionist one.²⁵

7. 7. DIFFERENT FORMULATIONS OR DIFFERENT INTERPRETATIONS? A REPLY TO AN OBJECTION

It may be objected that different formulations are equivalent in any important respect and the Newtonian and the Lagrangian articulations of mechanics are just different formulations of a single theory: they are *physically* and *metaphysically* equivalent. Most physics textbooks promote such an idea.²⁶ However, this claim is not always true. Different formulations may imply different mathematical structures or different physical or metaphysical pictures of the world.

To clarify this point, let us consider the case of different formulations of classical mechanics. The common view is that the Newtonian, Lagrangian, and Hamiltonian formulations are mutually equivalent because their dynamical equations are inter-derivable. However, Wharton points out that this argument requires “a bit of circular logic” (Wharton 2015b: 182) because dynamical equations are not everything.

A typical argument for schema-equivalence is to use Fermat’s principle [the corresponding Lagrangian-style law] to derive Snell’s law of refraction, the corresponding Newtonian-style law. In general, one can show that action minimization always implies such dynamic laws. . . . But a dynamical law is not the whole Newtonian Schema. . . . [T]he input and output steps differ: Snell’s law takes different inputs than Fermat’s Principle and yields an output (the final ray position) that was already constrained in the action minimization. (Wharton 2015b: 182)

While Snell’s law takes the angle of incidence as input, the angles of incidence and reflection are not the inputs of Fermat’s principle but its outputs. Therefore,

²⁵The term “holistic” can have a wide range of meanings even in physics, let alone philosophy (see Healey and Gomes 2022). Also, a distinction can be made between “global” (vs. local) and “holistic” (vs. reductionist) perspectives. Nevertheless, here we use “global” and “holistic” interchangeably. Also, we use both in the sense explained in Section 6, i.e., considering all degrees of freedom by applying the Lagrangian and obtaining the law governing the system by minimizing the action. This is what the Lagrangian schema entails and what Wharton has in mind when speaking of the “holistic” point of view. Including more than that requires additional arguments that go beyond the scope of this article.

²⁶See, for example, (H. Goldstein, Poole, and Safko 2001: 334) and (Thornton and Marion 2004: 257–258).

the inter-derivability of the dynamical equations of the Newtonian, Lagrangian, and Hamiltonian formulations does not require that these formulations have the same calculation procedure.

Is there a deeper difference in the mathematical structure or the underlying metaphysics of these formulations? The answer is affirmative. Jill North (2009, 2021a, 2021b) discusses some mathematical, physical, and metaphysical differences of different formulations of classical mechanics.²⁷ She points out that there are disagreements among the Newtonian, Lagrangian, and Hamiltonian formulations over what is *fundamental*, that is, what is real at the fundamental level of the reality. One way of expressing these differences is as follows:²⁸

- According to the Newtonian formulation, the world at its fundamental level contains particles and forces between particles, which are basic and irreducible concepts.
- According to the Lagrangian formulation, the world at its fundamental level contains particles and energies, and the difference between kinetic and potential energies is the fundamental concept. The real dynamical properties of particles are configuration-like quantities. Momentum-like quantities are defined as the first derivative of configuration-like quantities with respect to time. Force is not a fundamental concept and is derivable from energy.
- According to the Hamiltonian formulation, the world at its fundamental level contains particles and energies, and the total energy is the fundamental concept. The real dynamical properties of particles are both configuration-like and momentum-like quantities, which are determined independently of each other. Again, force is not a fundamental concept and is derivable from energy.

Also, it is noteworthy that the Newtonian, Lagrangian, and Hamiltonian formulations are not *empirically* equivalent. For example, Xin Wu et al. (2015) and Rongchao Chen and Wu (2016) demonstrate that the Lagrangian and Hamiltonian

²⁷Also see (Curiel 2014). While North believes that the formulation of the theory in which to best represent the classical world is the Hamiltonian formulation, Curiel believes that “classical systems evince the structure intrinsic to Lagrangian mechanics, nothing more and nothing less” (Curiel 2014: 294).

²⁸Also see (Baez and Wise 2019) and (Lanczos 1952).

formulations are *not* empirically equivalent but are only approximately related in the post-Newtonian regime due to the truncation of higher-order terms. In some cases, the Lagrangian approach shows chaos (see Levin 2003), while the corresponding Hamiltonian approach does not (see Königsdörffer and Gopakumar 2005 and Gopakumar and Königsdörffer 2005).

Therefore, the Newtonian, Lagrangian, and Hamiltonian formulations could be different in their mathematical structure, physical content, and metaphysical assumptions. In general, different formulations may imply different mathematical structures or different physical or metaphysical pictures of the world.

8. THE ADVANTAGES OF THE LAGRANGIAN SCHEMA OVER THE NEWTONIAN SCHEMA

As mentioned in section 6, the Lagrangian schema enjoys a global and holistic point of view. This encouraged Wharton (2013a, 2015b) to claim that the metaphysical picture of the quantum world that the Lagrangian schema requires does not suffer from the problems that the Newtonian schema faces and is the natural generalization of the picture that the Newtonian schema provides. In addition, he believes that although our best reason to choose the Lagrangian schema can be found in the quantum realm, we can also find evidence of the superiority of the Lagrangian schema in the classical regime.²⁹ We do not intend to evaluate his claims. Instead, we return to Smolin's criticisms and argue that some of Smolin's criticisms of the Newtonian schema do not apply to the Lagrangian schema.

Smolin's criticisms of the Newtonian schema can be summarized as follows: applying the Newtonian schema to the world as a whole leads to

- the cosmological fallacy,
- the undesirable result of the unreality of time,
- the undesirable result of the existence of the multiverse,
- the cosmological dilemma, and

²⁹Chen and Goldstein (2022) also mention a similar point. They believe that both dynamic production and the Newtonian schema are problematic in the non-quantum regime. Also, Lanczos (1952: xxiv-xxv) argues that it is better to use the Lagrangian formulation for presenting general relativity. His argumentation can be employed with minor modifications to demonstrate the better applicability of the Lagrangian schema than the Newtonian to formulate and interpret relativity theory.

- questions that cannot be answered within the Newtonian schema.

Let us now examine in each case whether these criticisms apply to the Lagrangian schema.

The cosmological fallacy. As mentioned in Section 3, the cosmological fallacy has its roots in this false claim that “it is very likely that the Newtonian schema is successfully applicable to the world as a whole.” Smolin points out that this claim has no empirical support and is based on an incomplete analogy. The application of the Newtonian schema is allowed only *locally* and to the subsystems of the world. This is not the case for the Lagrangian schema. As mentioned in Section 6, the Lagrangian schema enjoys a global and holistic point of view and is, therefore, quite capable of being applied to the world as a whole. To do this, it is enough to remember the success with which it has been applied to the subsystems of the world and to be willing to use an analogy similar to the one we used in the case of the Newtonian schema. Therefore, the application of the Lagrangian schema to the world as a whole does not lead to the cosmological fallacy.

The unreality of time. As mentioned in Section 2, Smolin believes that the unreality of time is a *direct* consequence of the cosmological fallacy. If the application of the Lagrangian schema to the world as a whole does not lead to the cosmological fallacy, the unreality of time would not be the *direct* consequence of the application of the Lagrangian schema. This is progress, but it does not mean that the application of the Lagrangian schema guarantees the reality of time. The place of time in the Lagrangian schema universe needs further investigation. Moreover, our best theories about the fundamental level of physical reality, such as loop quantum gravity, of which Smolin is one of the founders and defenders, widely use the Lagrangian and Hamiltonian formulations. It is quite possible that these theories can be interpreted from the Lagrangian schema’s point of view. The time parameter is absent in these theories’ basic equations, which describe the fundamental level of reality, i.e., quantum spacetime.³⁰ Replacing the Newtonian schema with the Lagrangian schema may not help *in this regard*. Therefore, it seems that time can be considered unreal even in the Lagrangian schema, and time is unreal according to our best theories of quantum gravity. So, applying the Lagrangian schema does not guarantee the reality of time. This would not be desirable for Smolin, as the defender of temporal naturalism.

³⁰For an introductory and non-technical review of the theories of quantum gravity, see (Smolin 2001). For an introductory but technical review, see (Wallace 2000). For an introduction to the problem of time in quantum gravity, see (Weinstein and Rickles 2024) and (Wallace 2000).

Multiverse. As mentioned in Section 3, the invalid application of the Newtonian schema to the universe as a whole requires the (unjustified) belief in the multiverse. The question then arises: does applying the Lagrangian schema guarantee that the multiverse hypothesis will not be needed? The answer appears to be negative. If we consider different universes within a multiverse as obeying the same laws but starting with different initial conditions, as per Smolin, the only thing that changes in the Lagrangian schema is that different universes, while obeying the same laws, differ with respect to some different (temporally) *initial and/or final* conditions. The belief in the multiverse persists because different universes can be associated with different boundary conditions. Here, “boundary” refers to past as well as future conditions that shape the universe. In this view, the multiverse would consist of universes governed by the same Lagrangian, but differing in their *initial and/or final* conditions. This distinction arises because, while the Newtonian schema takes inputs that are only initial, the Lagrangian schema incorporates both initial and final inputs.³¹

The cosmological dilemma. As mentioned in Section 3, maximum accuracy is achieved when we take all the degrees of freedom in the world into account; or, equivalently, we expand the boundaries of the subsystem at study to include the entire world. Smolin warns us that in this situation we are left with only one system and only one possibility of measurement; so, the distinction between the law and the initial conditions collapses. This brings us to the cosmological dilemma, forcing us to either acknowledge the limitations of our current physical laws or abandon the very concept of a universally applicable law. Either way, we are not justified to apply that general law to the world as a whole. As mentioned in sections 6 and 7, the Lagrangian schema dispensed with this distinction from the outset. Within the Lagrangian schema, the input is not the initial conditions but a combination of the initial and subsequent states of the system. The same is true about its output. Therefore, the application of the Lagrangian schema to the world as a whole does not lead to the cosmological dilemma. The concept of the general law is defined *holistically* and *globally* according to the Lagrangian schema.

³¹Also, one could define a multiverse in which the universes differ not only in their initial and/or final conditions but also in their governing laws. See, for example, (Linde 2004), (Masoumi, Golshani, and Sheikh Jaber 2014), and (Masoumi 2014). In this case, different universes would also be associated with different Lagrangians.

But is this kind of “law” the one Smolin has in mind when talking about the inside-time laws of cosmology? Remember that Smolin defends temporal naturalism. The commonplace view is that laws of nature are timeless and immutable to change. In contrast, temporal naturalism holds that laws of nature evolve in time.³² This evolution is under “cosmological natural selection,” a similar notion to the one employed in evolutionary biology.³³ According to Smolin, the laws governing the universe should be prone to change.³⁴ The laws of the Lagrangian schema do not seem like this as they do not evolve in time. Rather, they seem more like the outside-of-time laws of timeless naturalism: they determine the world in an all-at-once manner, so they are immutable to change over time.

This intensifies the tension between the Lagrangian schema and the belief in the reality of time because Smolin believes that

[i]f the laws are the result of evolution in time, then time is prior to the laws because if the laws can change in time, then time is something more fundamental: it doesn’t emerge from a particular law. And this leads to the view that time really is so deep, so fundamental, that it’s even prior to the regularities that we characterize as laws of nature. (Burton H. (ed.) 2021, part II)

But, as mentioned above, applying the Lagrangian schema does not guarantee the reality or fundamentality of time.

The unanswerable cosmological questions. According to what was said in the previous paragraph, it is clear that the questions mentioned in Section 4 will not arise by following the Lagrangian schema. This is the consequence of rejecting mechanical or computational philosophy and, therefore, rejecting the cosmic computer. However, it is not clear that other unanswerable questions will not arise when following the Lagrangian schema. It is not surprising if, within the

³²Smolin believes that the evolution of laws is the main advantage of temporal naturalism and a falsifiable scientific thesis that makes temporal naturalism more aligned with scientific inquiry. He states that “temporal naturalism has a much larger range of empirical adequacy than its rivals because it alone allows a conception of laws which can evolve in time. This . . . is necessary if we wish the choice of laws to be explicable on the basis of hypotheses that are falsifiable by the results of doable experiments” (Smolin 2015: 86–87). Also, “by making laws evolvable in ways that are testable, temporal naturalism makes cosmology more scientific than timeless naturalism, which by keeping laws as timeless and immutable also puts them beyond explanation by means of testable hypotheses” (Smolin 2015: 97).

³³For more detail, see (Smolin 1992, 1997, 2013a).

³⁴According to Smolin, this claim is a scientific claim that can be falsified, for example, if it turned out that there exist neutron stars with masses, say, 2.5 times the mass of the sun somewhere in the universe. For more details, see (Burton H. (ed.) 2021, part V).

framework of the Lagrangian schema, questions such as “why does this particular general law and not another govern the world in an all-at-once manner?” are meaningful and yet unanswerable. It is not unlikely that the Lagrangian schema is in a similar situation to the Newtonian schema in this respect; however, it is doubtful or debatable that this would be a weakness for either of the two. One might justifiably believe that these questions are originally philosophical. It may even be claimed that such questions go beyond the scope of naturalism and answering them requires taking a non-naturalistic direction.³⁵

To summarize, unlike the Newtonian schema, the use of the Lagrangian schema does not lead to the cosmological fallacy and dilemma. However, it is doubtful that the Lagrangian schema has any *significant* advantage over the Newtonian schema from Smolin’s point of view in the other cases.

9. CODA

Current physics and modern cosmology widely use the Newtonian schema, in which a general law is sharply distinguished from the initial conditions. This schema has been used to describe and explain the evolution of physical systems as well as the world as a whole over time. According to the Newtonian schema, the universe is like a computer whose state at every moment is determined by receiving initial conditions as inputs and applying the general laws as an algorithm. Smolin and Wharton believe that this anthropocentric picture of the world as a cosmic computer has led to a crisis in scientific explanation. One suggestion is to replace the Newtonian schema with the Lagrangian schema. The Lagrangian schema enjoys a holistic and global point of view, according to which the system at study or the world as a whole is examined in an all-at-once manner rather than as the time evolution of a timeless part. According to the Lagrangian schema, the cosmic computer does not exist. Although the Lagrangian schema does not suffer from some of the problems of the Newtonian schema, it still faces some similar challenges. For example, even if one uses the Lagrangian schema, time can still be unreal, and the multiverse can be existent. However, Smolin considers denials of both the unreality of time and the existence of the multiverse as the two pillars of “temporal naturalism,” the doctrine that he believes is the way out

³⁵Recall that Smolin advocates a particular kind of naturalism, namely temporal naturalism, thus already avoiding such an objection.

of the crisis. So, the Lagrangian schema, proposed by Wharton as an alternative to the Newtonian schema, cannot be so favorable to Smolin.

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